Lecture 13:
Hard-to-Borrow Securities

Marco Avellaneda

G63.2936.001

Spring Semester 2009

Hard-to-Borrow Stocks: Price dynamics and Option Valuation

Marco Avellaneda, New York University
Mike Lipkin, Columbia University and Katama Trading

RiO, Hotel do Frade, November 26 2008

Hard-to-borrow stocks: A new focus on an ``old'' problem?

- Borrowing stocks is necessary for short-selling (delivery in T+2 !)
- Availability of stocks for borrowing is often limited and is variable
- Restrictions on short-selling vary strongly in time
- Adam's Not-so-Invisible Hand: In September 2008 the SEC restricted for 1 month shorting in ~800 stocks (mostly financials)
- Regulation SHO: stocks must be ``located'' before they are shorted
- Option market-makers are generally exempt from SHO but are subjected to **buy-ins** by their clearing brokers
- New solutions needed to achieve more transparency in stock-lending

Characteristics of HTB stocks

- Nominal Put-Call Parity does not hold (what to do about classical RN pricing?)
- Increased volatility
- Unusual pricing of vertical spreads (Put spreads/ call spreads)
- Short Squeezes
- Financing costs imply reduced, even negative, rates for shorting

Bye-bye, Put-Call Parity

	Date	1/8/2008	EXP	1/1/2009								
calls	DNDN	5.81		puts							d_htb [={p_pop-c_pop+rKt-dSt}/(St)]	
bestbid	bestoffer	IVOL	strike	bestbid	bestoffer	IVOL	Days	Pmbbo	Cmbbo	Ppop	Cpop	d_htb
3.55	3.7	83%	2.5	0.57	0.58	121%	373	0.575	3.625	0.575	0.315	5.81%
1.853	3.2	129%	5	2.65	2.67	162%	373	2.66	3.1	2.66	2.29	9.12%
2.48	2.64	132%	7.5	4.75	4.85	172%	373	4.8	2.56	3.11	2.56	13.60%
2.11	2.16	132%	10	6.85	7	173%	373	6.925	2.135	2.735	2.135	15.91%
	1.88	134%	12.5	8.95	9.1	169%	373	9.025	1.865	2.335	1.865	15.24%
1.43	1.6	129%	15	11.10	11.45	169%	373	11.275	1.515	2.085	1.515	18.39%
1.35	1.45	132%	17.5	13.30	13.65	166%	373	13.475	1.4	1.785	1.4	16.80%
1.05	1.14	126%	20	15.25	15.6	151%	373	15.425	1.095	1.235	1.095	14.22%
0.86	1.02	124%	22.5	17.50	17.85	147%	373	17.675	0.94	0.985	0.94	14.12%
0.7	0.8	120%	25	19.75	20.15	143%	373	19.95	0.75	0.76	0.75	15.03%
0.47	0.58	116%	30	24.45	25.25	149%	373	24.85	0.525	0.66	0.525	20.08%
0.3	0.4	111%	35	29.25	30.1	149%	373	29.675	0.35	0.485	0.35	23.05%
0.15	0.28	105%	40	34.05	34.75	140%	373	34.4	0.215	0.21	0.215	23.70%
0.11	0.18	102%	45	38.95	39.4	-9999%	373	39.175	0.145	-0.02	0.145	24.09%

Ppop=mbboP-max(K-S,0)

Cpop=mbboC-max(S-K,0)

Dendreon (DNDN)

Dendreon Corp.	$ 4.51
DNDN	-0.14

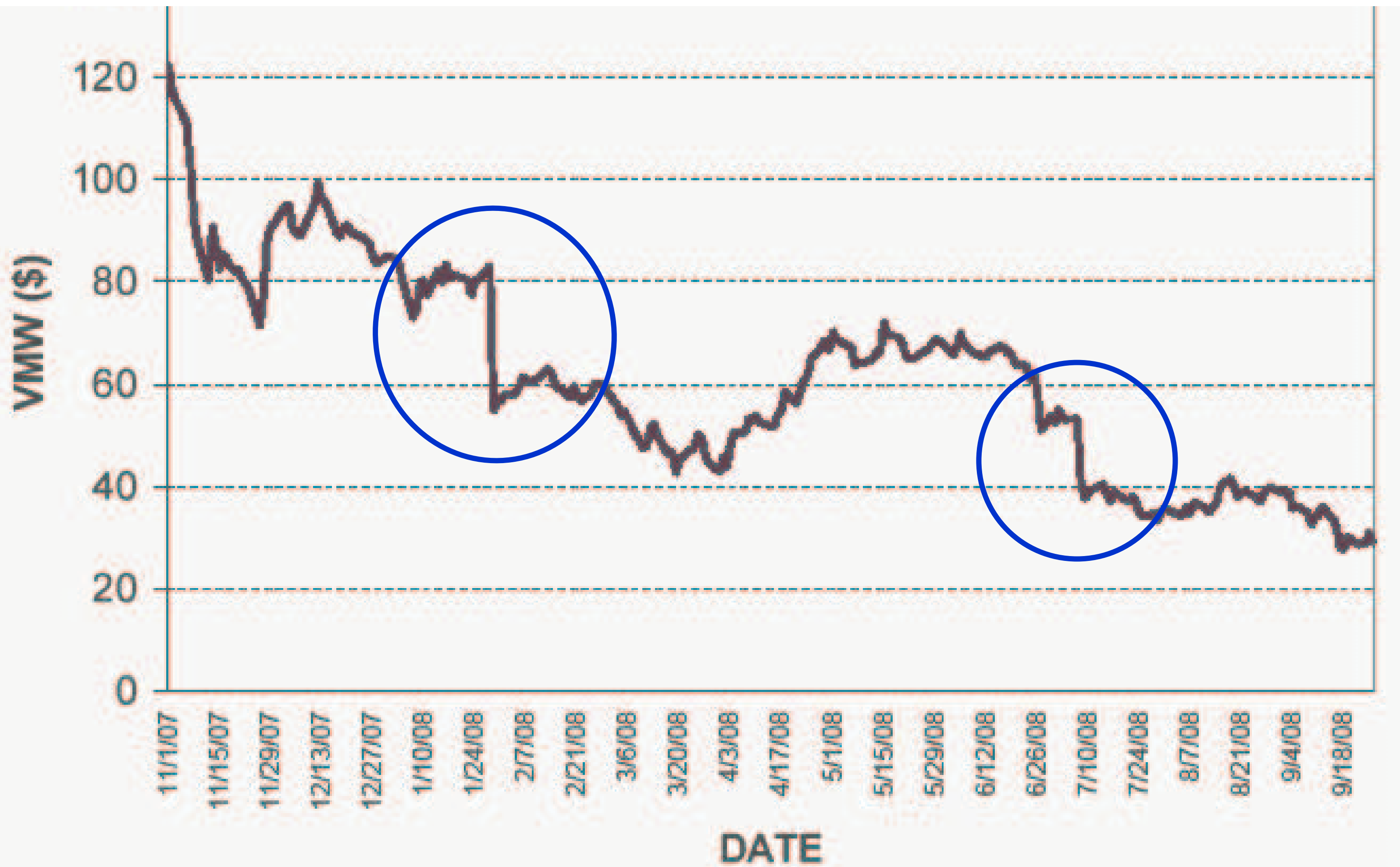
Short Interest (Shares Short)	24,337,600
Days To Cover (Short Interest Ratio)	18.6
Short Percent of Float	27.29 %
Short Interest - Prior	25,076,900
Short % Increase / Decrease	-2.95 %
Short Squeeze Ranking™	-104

Short rate in October 2008= 19.7% !

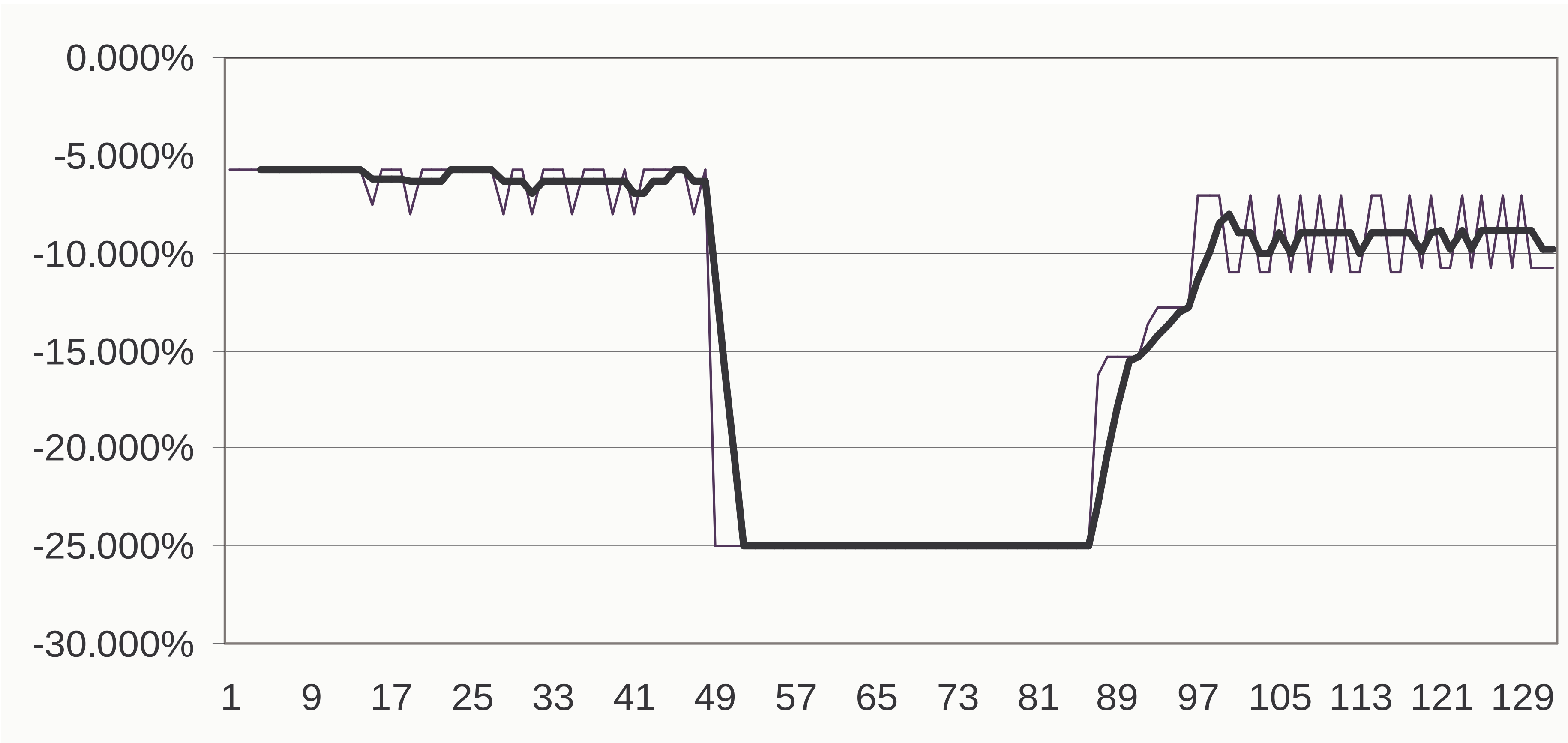
Krispy Kreme Donuts (KKD)

From 2001 to 2004, Krispy Kreme was extremely hard to borrow, with frequent buy-ins. The candlesticks show the stock was very volatile and high-priced reaching $200 (unadjusted).

VMWare Nov 07 – Sep 08

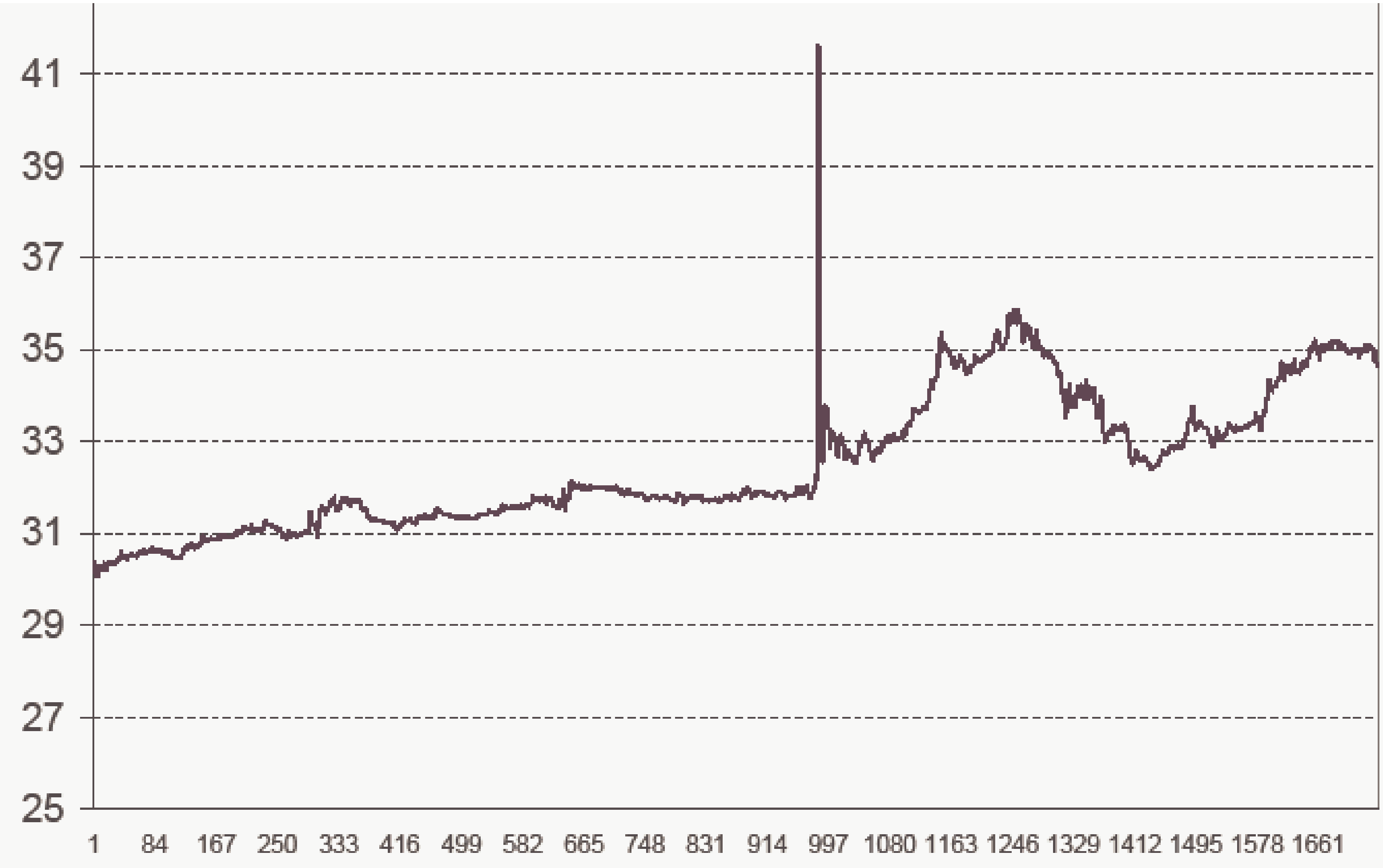


VMWare Short Rate (1/2007-8/2008)

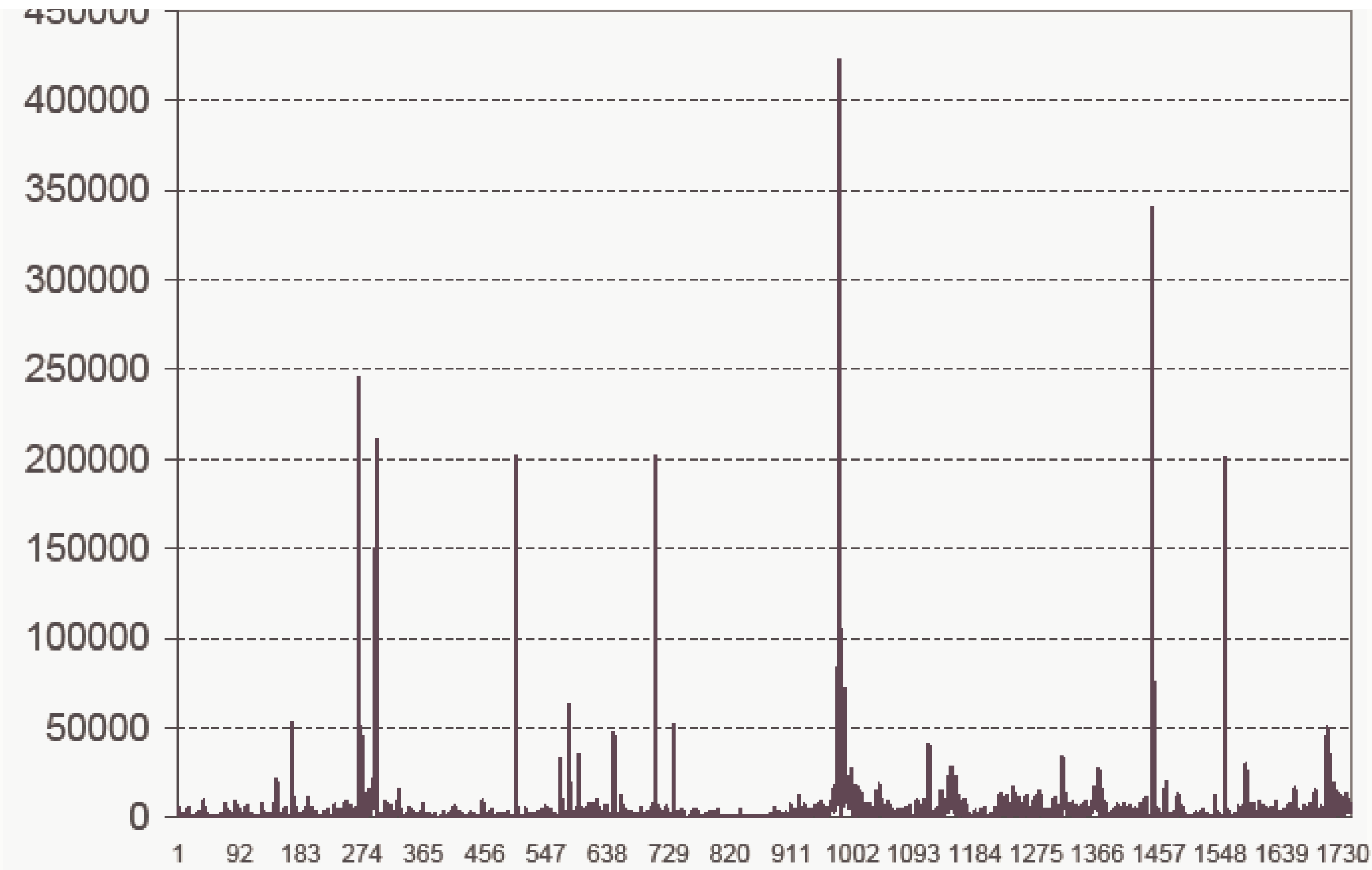


If you short, you don’t receive interest on cash. Instead, you pay up to 25%

Interoil Corporation (IOC)



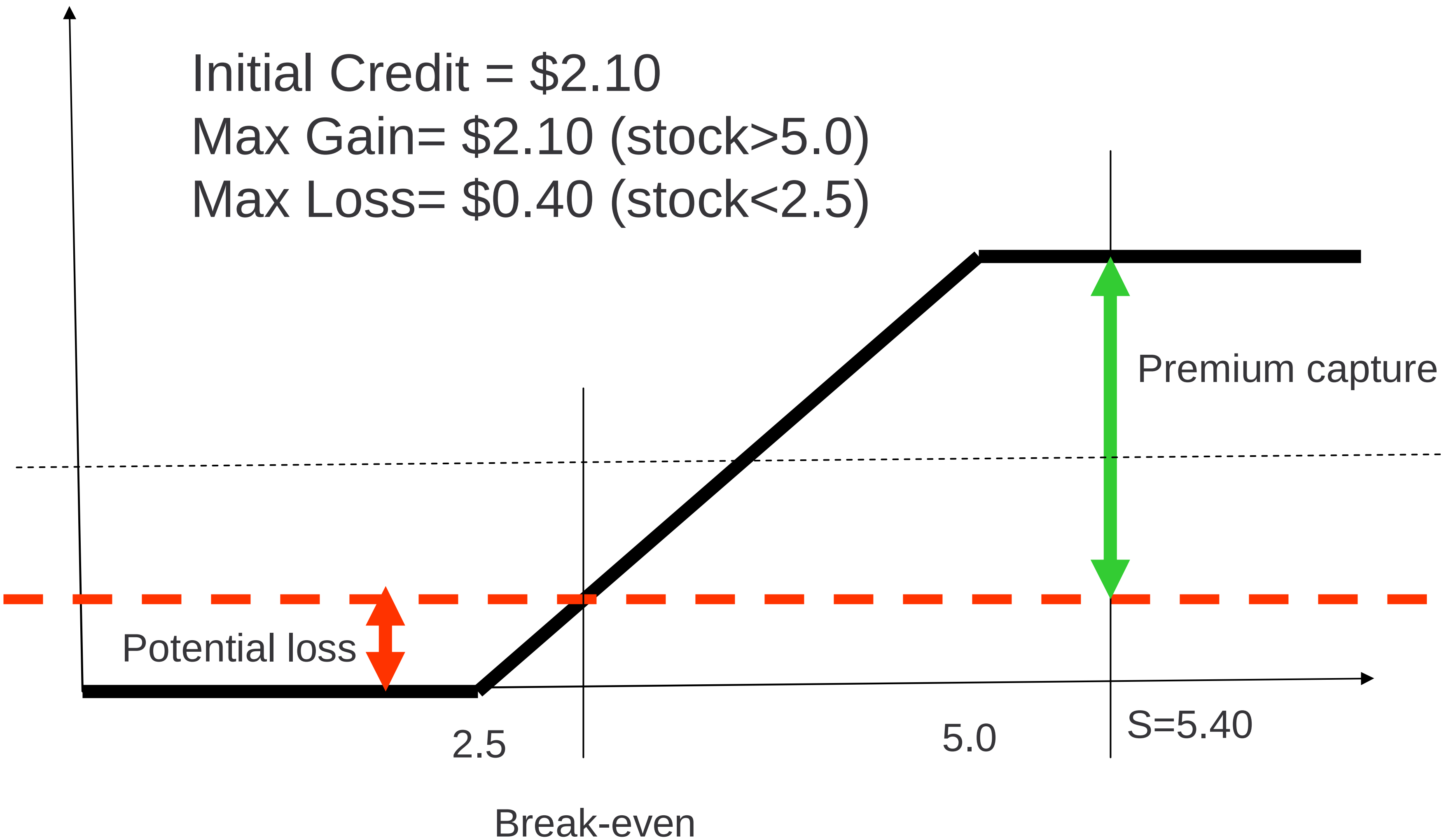
IOC Traded Volume



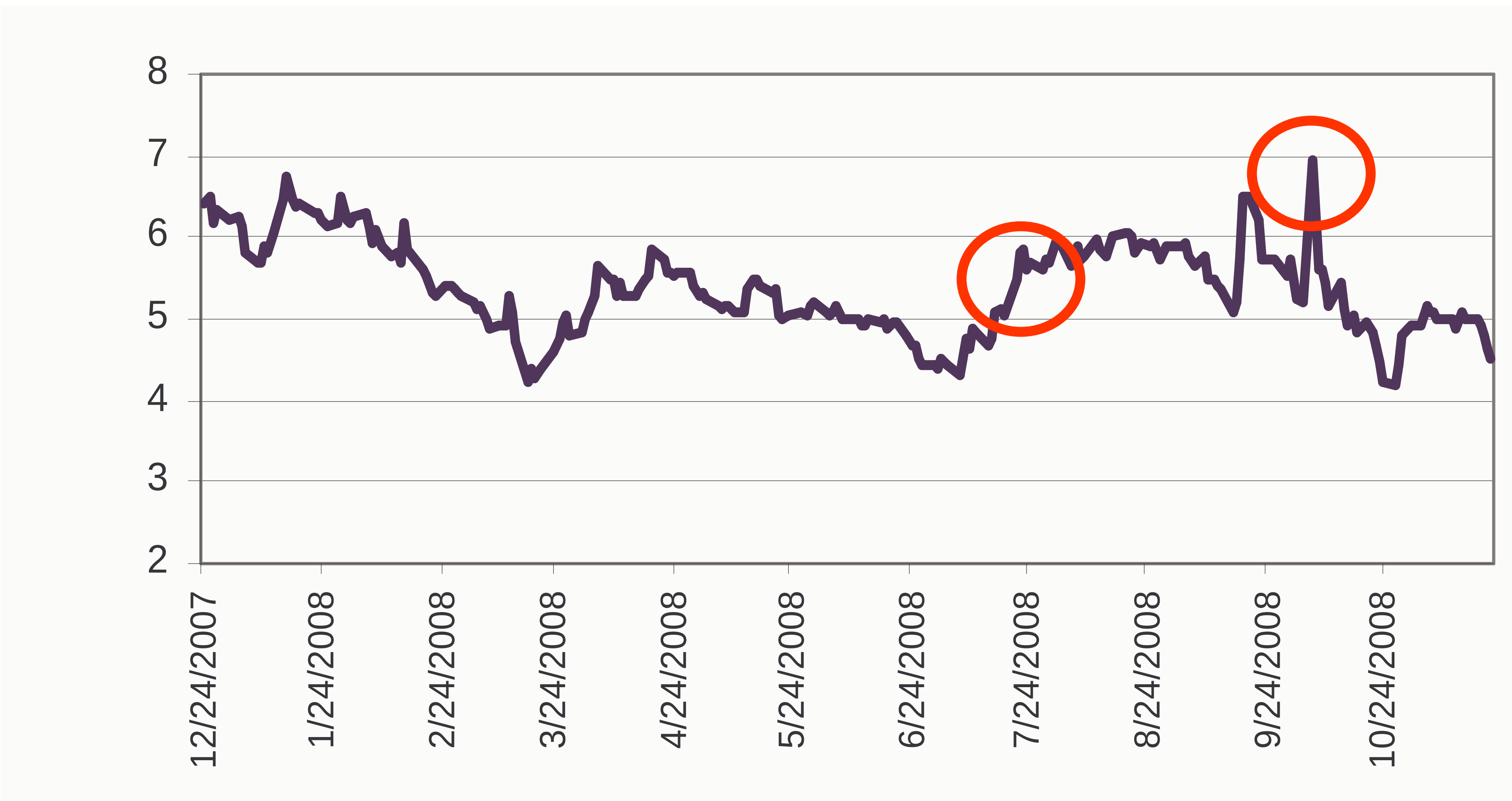
Unusual Verticals -- Valuation in July 2008

DNDN = $ 5.40
Jan 09 2.5P = $ 0.45
Jan 09 5.0P = $ 2.55
2.5/5.0 Put Spread = $2.10

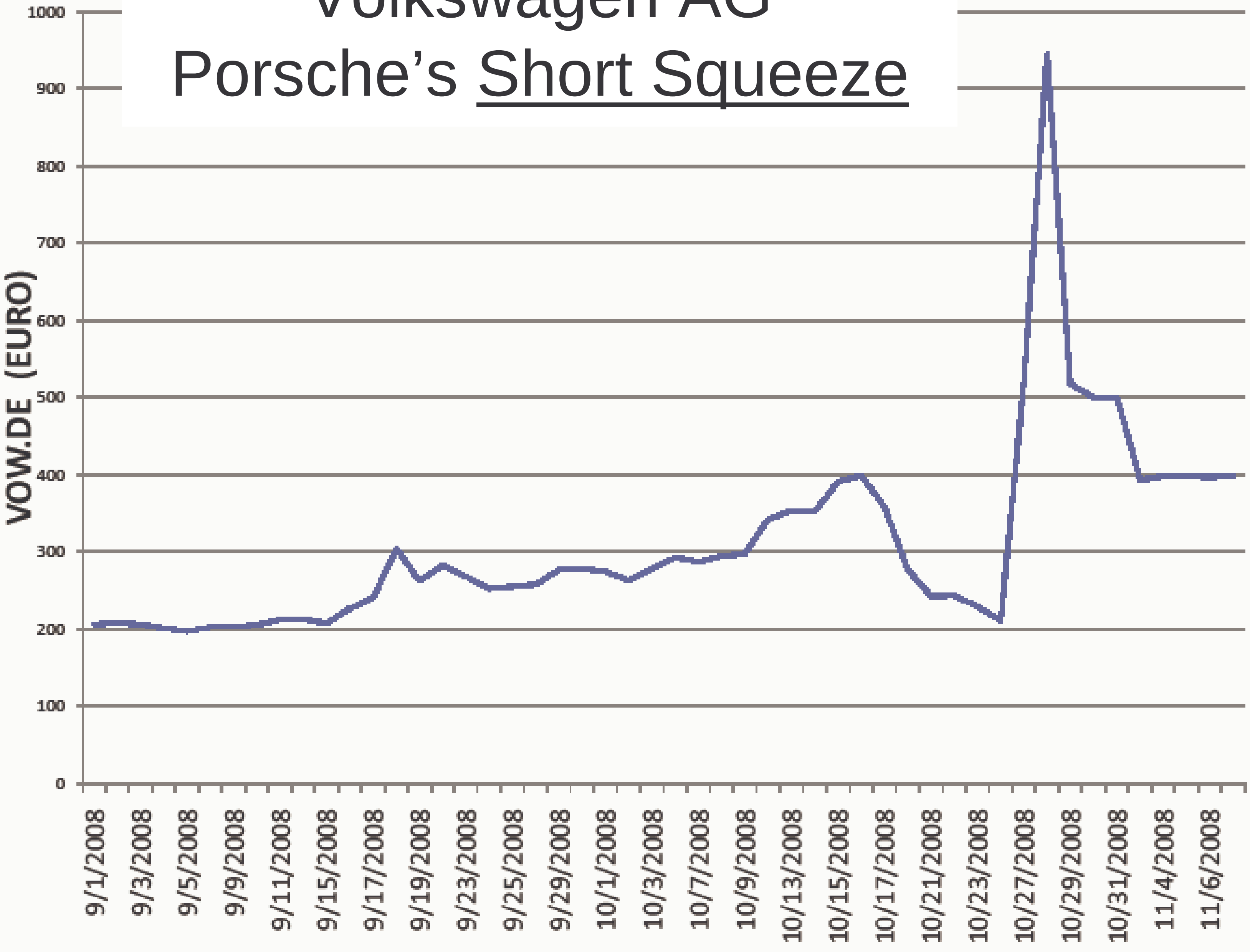
Initial Credit = $2.10
Max Gain= $2.10 (stock>5.0)
Max Loss= $0.40 (stock<2.5)



DNDN (Dec 07 to 11/08)



Volkswagen AG
Porsche's Short Squeeze



Porsche Long Calls; HF short stocks

Citicorp (C)

Citigroup Inc.	$ 3.77
C	-0.94

Short Interest (Shares Short)	138,025,500
Days To Cover (Short Interest Ratio)	1.0
Short Percent of Float	2.70 %
Short Interest - Prior	116,765,900
Short % Increase / Decrease	18.21 %
Short Squeeze Ranking™	-2

October 2008 borrow rate=-5.6%

Goldman Sachs (GS)

The Goldman Sachs Group Inc.	$ 53.31
GS	1.31

Short Interest (Shares Short)	9,027,400
Days To Cover (Short Interest Ratio)	0.4
Short Percent of Float	2.50 %
Short Interest - Prior	7,970,200
Short % Increase / Decrease	13.26 %
Short Squeeze Ranking™	-1

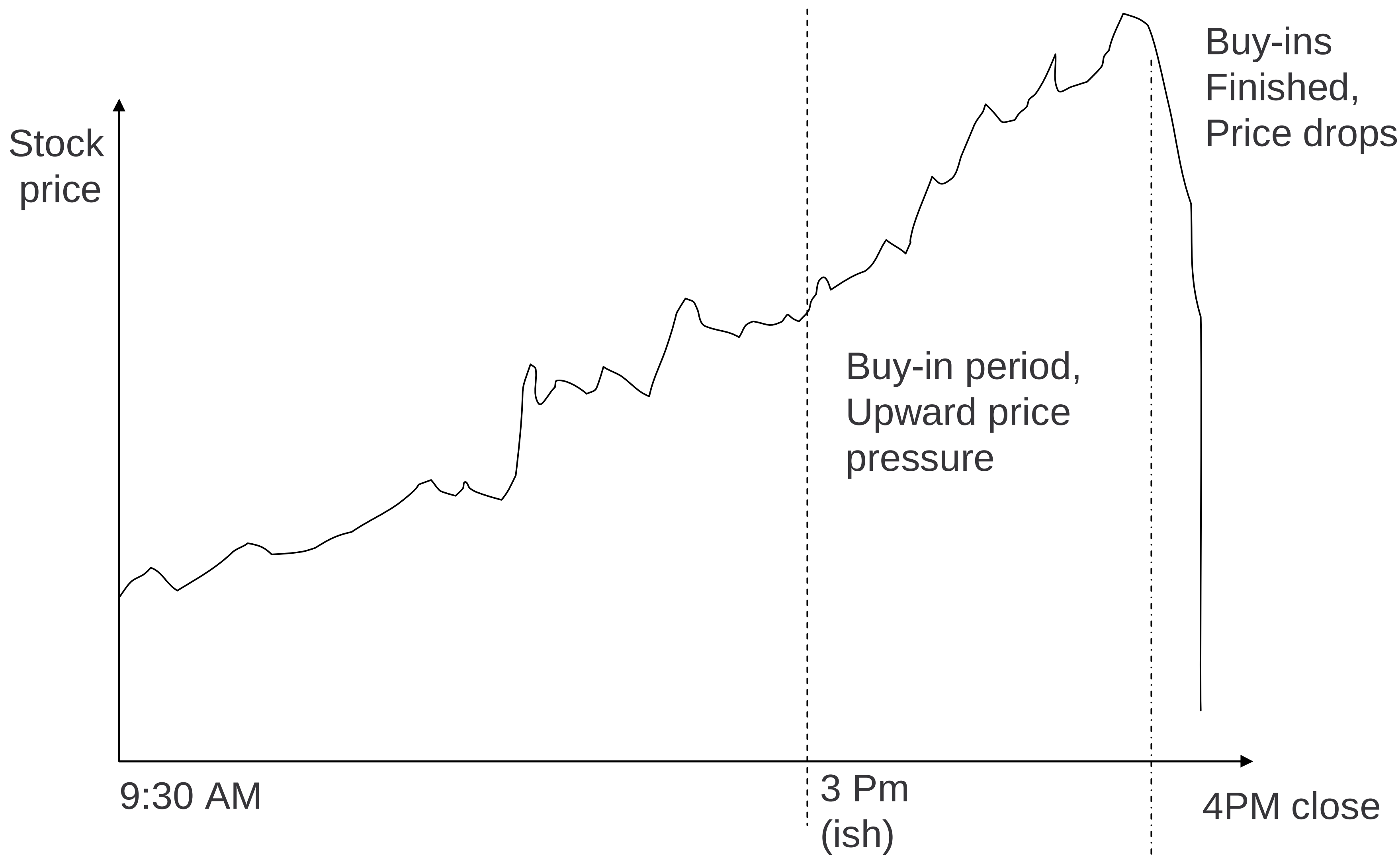
October 2008 borrow rate= -0.1 %

General Motors Corp. (GM)

General Motors Corp.	$ 3.06
GM	0.18

Short Interest (Shares Short)	102,575,700
Days To Cover (Short Interest Ratio)	4.3
Short Percent of Float	18.10 %
Short Interest - Prior	93,598,400
Short % Increase / Decrease	9.59 %
Short Squeeze Ranking™	-70

Buy-ins: A Market-Maker's Perspective



The Model: series of buy-ins with stochastic buy-in rate

$$\begin{cases} \dfrac{dS}{S} = \sigma dZ + \gamma\lambda dt - \gamma dN_\lambda(t) \\[2em] d\ln\lambda = \kappa dW + \alpha\left(\overline{\ln\lambda} - \ln\lambda\right)dt + \beta\dfrac{dS}{S} \end{cases}$$

$\lambda =$ Buy-in rate $\qquad N_\lambda(t) =$ Poisson counter, intensity λ

$\gamma =$ Scale parameter

$\beta =$ Coupling constant

$\alpha =$ Mean-reversion rate

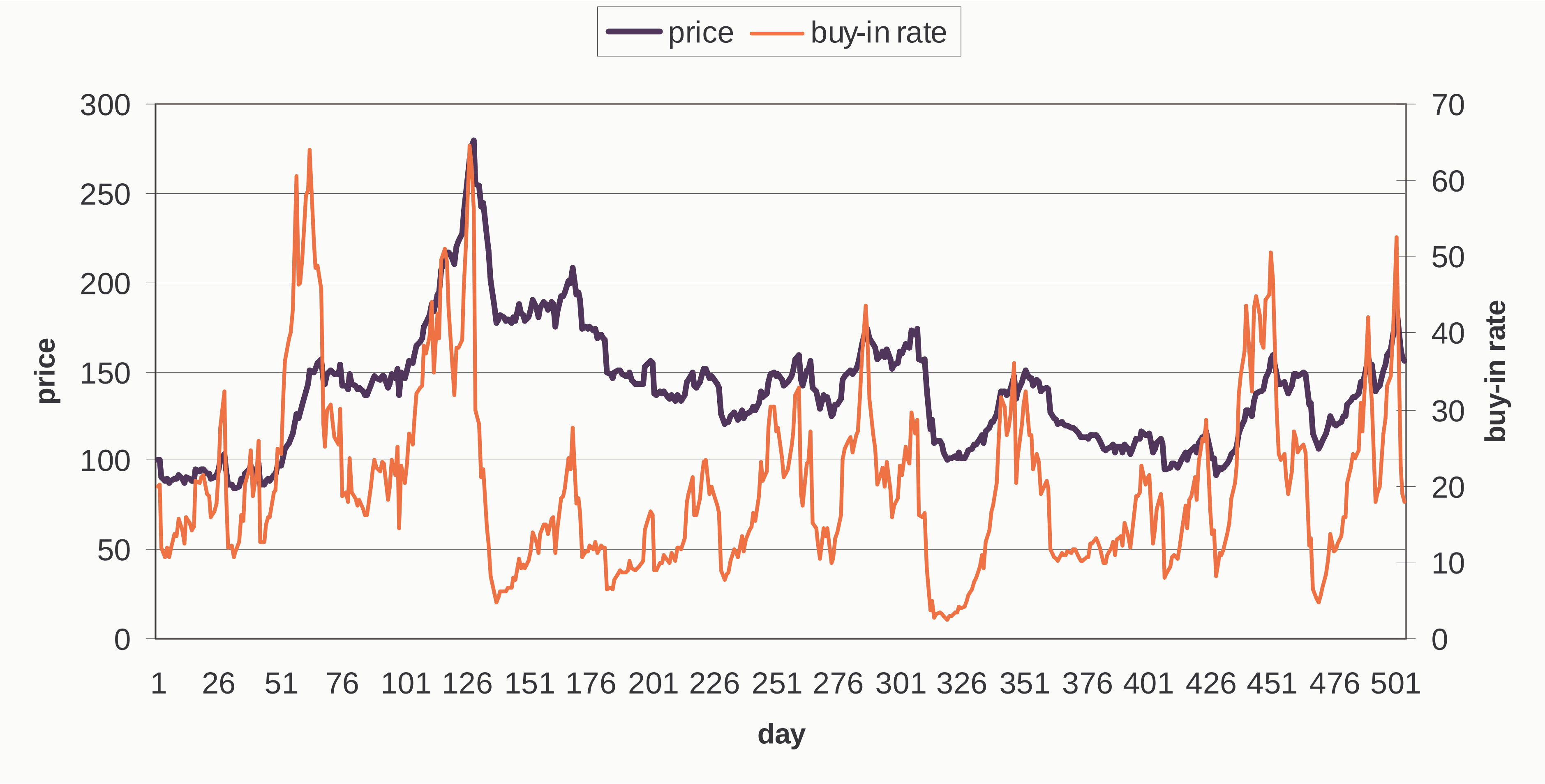
``Solution'' of the Model

$$d\ln\lambda = \varpi dZ' + \alpha\left(\overline{\ln\lambda} - \ln\lambda\right)dt + \beta\gamma\left(\lambda dt - dN_{\lambda}(t)\right)$$

$$S_t = S_0 M_t e^{\int_0^t \gamma\lambda_s ds}\left(1-\gamma\right)^{\int_0^t \gamma dN_{\lambda_s}(s)}$$

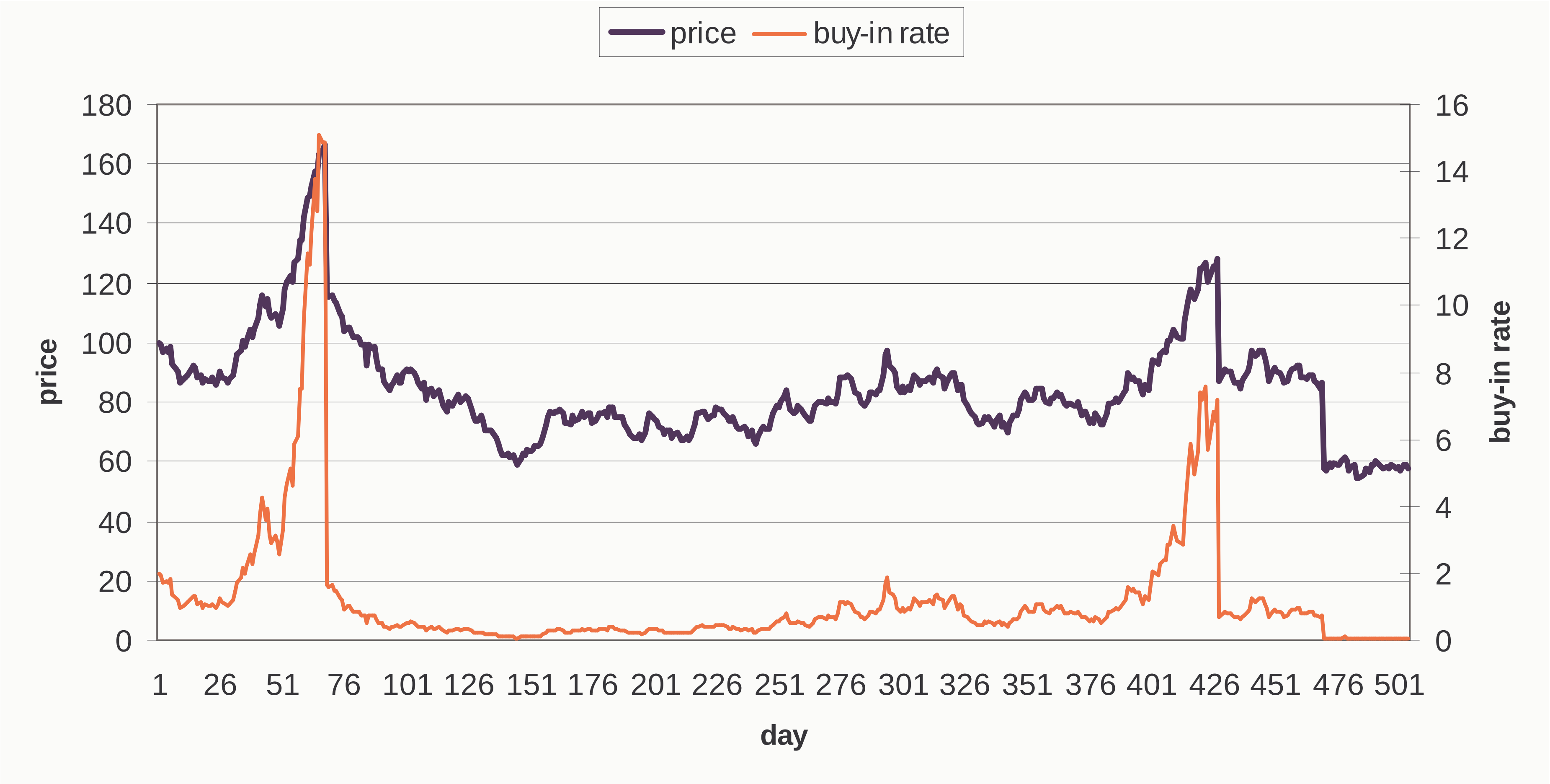
$$M_t = e^{\sigma W_t - \frac{\sigma^2 t}{2}}$$

It can be shown that the price is a ``local martingale''.



Initial Lambda=20, effective dividend=10%, Gamma=0.05

Kappa=1%, Beta=5, alpha (mean rev for BIR)=1yr



Initial Lambda=2, effective dividend=1%, Gamma=0.05

Bursting behavior due to sporadic increase in buy-in rate.

Implications for option pricing

Expected loss per share due to the **lack of deltas after buy-in:**

$$loss = \begin{cases} \gamma S, & \text{with prob } \lambda dt \\ 0, & \text{with prob } 1-\lambda dt \end{cases}$$

$$E\left(loss \mid \lambda, S\right) = \gamma\lambda S dt$$

Agents would pay this to ensure that the short stock is not removed.

Conclusion: there is a convenience yield for holding stock (you can lend it) and the ``fair dividend rate'' (stochastic) is

$$d_t = \gamma\lambda_t$$

Risk-neutral measure

$$\frac{dS}{S} = \sigma dW + \gamma\left(\lambda dt - dN_{\lambda}(t)\right) - \gamma\lambda dt + rdt$$
$$= \sigma dW - \gamma dN_{\lambda}(t) + rdt$$

$$S_t = S_0 e^{\sigma W_t - \sigma^2 t/2 + rt} \cdot (1-\gamma)^{\int_0^t dN_{\lambda_s}(s)}$$

$$E(S_t) = S_0 e^{rt} E\left\{ e^{-\gamma \int_0^t \lambda_s ds} \right\}$$

Implications for Option Pricing

$$\text{Forward Price}\,(T) = S_0 e^{rT} E\left\{ e^{-\gamma\int_0^T \lambda_s ds} \right\}$$

$$\text{Put}(S,K,T) - \text{Call}(S,K,T) = K \cdot e^{-rT} - S \cdot E\left\{ e^{-\gamma\int_0^T \lambda_s ds} \right\}$$

$$d_{eff}(T) = -\frac{1}{T}\ln E\left\{ e^{-\gamma\int_0^T \lambda_s ds} \right\}$$

Term structure of div rates

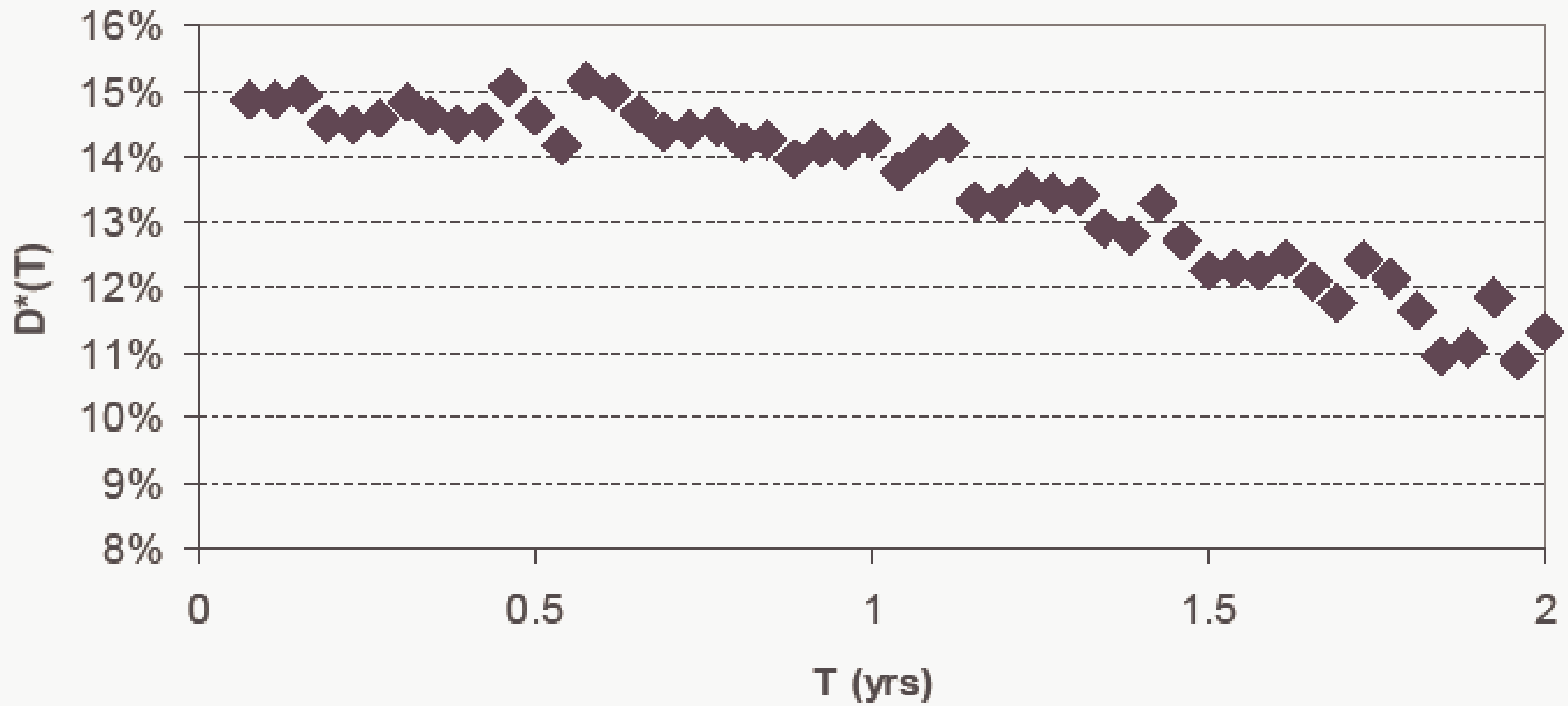
Implied Dividend Rate

$$(\text{ATM Put}) - (\text{ATM Call}) = Ke^{-rT} - Se^{-D_{imp}(T)T}$$

$$D_{imp}(T) = -\frac{1}{T}\ln\left(\frac{(\text{ATM Put}) - (\text{ATM Call}) - Ke^{-rT}}{S}\right)$$

The model gives a term-structure of effective dividends based on the anticipations for **hard-to-borrowness** (specialness) of the stock in the future

Term-structure of implied dividends



Option Pricing

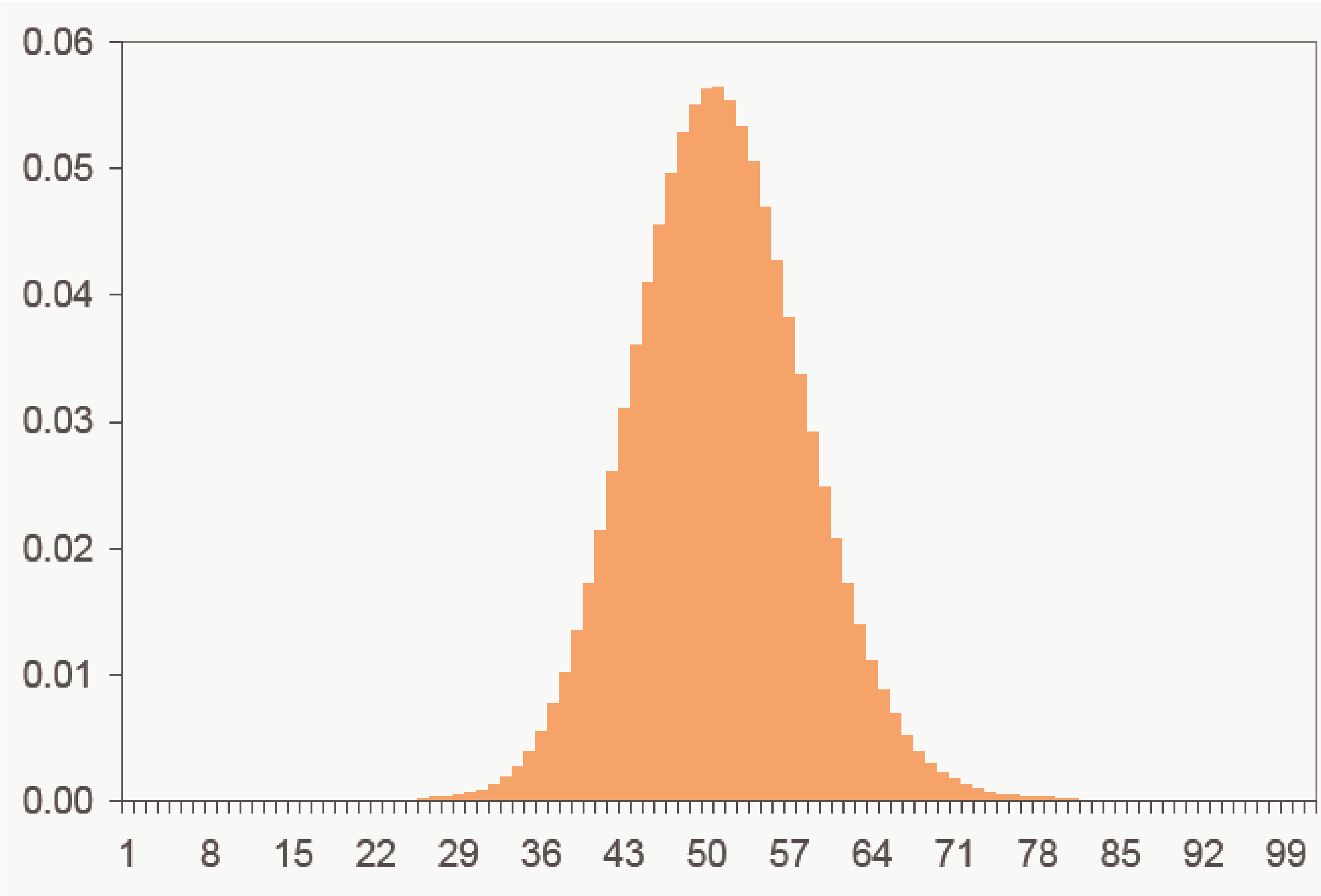
European options

$$\text{Define}: \qquad \Pi(n,T) = \Pr\left\{ \int_0^T dN_{\lambda_t}(t) = n \right\}$$

$$= \mathrm{E}\left\{ \frac{\left(\int_0^T \lambda_t dt \right)^n}{n!} e^{-\int_0^T \lambda_t dt} \right\}$$

$$\boxed{Call(K,T) = \sum_{n=0}^{\infty} \Pi(n,T) \cdot BSCall\left(S(1-\gamma)^n, T, K, r, \sigma\right)}$$

Poisson Weights for Option Pricing



American Options

- Need a 2D lattice method, or LSMC, to price
- Sometimes, it is optimal to exercise American Calls (even if there are no actual dividend payments)
- Deep ITM calls need to be hedged with a lot of short stock. The advantages of holding a synthetic puts may be compensated by the risk of buy-ins or negative carry for holding stock
- This is clearly confirmed by market observations

Dendreon Corp. (DNDN) Jan 09

Call early exercise **Put early exercise**

VMWare: Implied Dividend

Theoretical Implied Dividend (Calls have early exercise)

Extracting HTB Value from Spread Trades with Leveraged ETFs

$$S_t:\ \mathrm{IYF},\quad \mathrm{I\text{-}Shares\ Dow\ Jones\ U.S.\ Financials}$$

$$L_t^+:\mathrm{UYG},\ \mathrm{double\ long\ financial\ ETF}$$

$$L_t^-:\ \mathrm{SKF},\ \mathrm{double\ short\ financial\ ETF}$$

Managers of short-leveraged ETFs must borrow shares of the underlying index , incurring an additional cost.

This cost is stochastic, it depends on how difficult it is to borrow the under-lying securities

Leveraged ETF Spreads

$$\frac{dL_t^+}{L_t^+} = 2\frac{dS_t}{S_t} - rdt - fdt$$

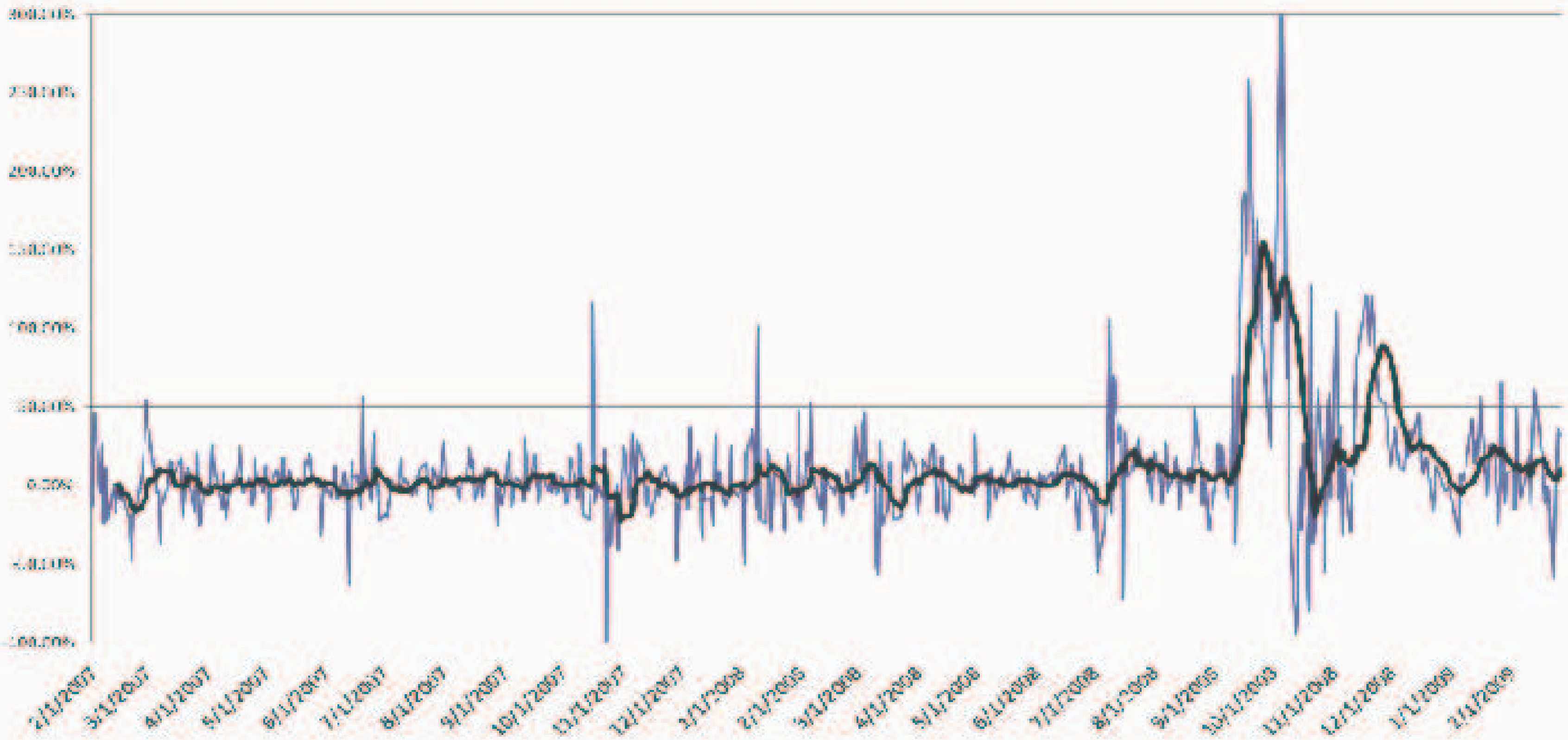
$$\frac{dL_t^-}{L_t^-} = -2\frac{dS_t}{S_t} + rdt + 2(r - \delta\lambda_t)dt - fdt$$

$$\frac{dL_t^+}{L_t^+} + \frac{dL_t^-}{L_t^-} = 2(r - f)dt - 2\delta\lambda_t dt$$

$$\delta\lambda_t dt = -\frac{1}{2}\left(\frac{dL_t^+}{L_t^+} + \frac{dL_t^-}{L_t^-}\right) + (r - f)dt$$

Short-short position in UYG and SKF pays the hard-to-borrowness

Evolution of the short SKF, short UYG spread, Feb 2007- Feb 2009



Conclusions

- HTB stocks and their options are interesting!
- Options on HTBs present breakdown of nominal Put-Call Parity
- Puts and Calls are`` in equilibrium'', but we must anticipate the cost of carry, or convenience yield
- Introduced a model for the fluctuations of prices based on an additional factor, the buy-in rate
- Model explains the term-structure of implied dividends
- American calls have optimal early exercise
- Generalization to **impossible to short stocks** (China, HK. Financials in Sep 2008)