Reflections on the Logic of the Ontological Argument*

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Introduction

Our 1991 paper on the logic of the ontological argument contained an analysis of the structure of Anselm's argument for the existence of God. We showed that there is a valid argument for God's existence in Proslogion II. However, in that paper, we deliberately decided not to include a discussion and analysis of the soundness of the argument. In these afterthoughts, we shall take up this question. We plan to argue for the following:

1. Anselm's argument for Premise 1 is not valid. This casts doubt on the truth of Premise 1 of the ontological argument.

2. If Premise 1 is revised so as to be clearly true, and the rest of the ontological argument is modified so as to preserve validity with the revised Premise 1, then the resulting argument is sound but doesn't have the conclusion that Anselm wishes to establish.

Our analysis in what follows appeals to the theory of abstract objects (Zalta 1983) and to the distinction between exemplifying and encoding a

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property fundamental to that theory. Zalta 2004 uses this distinction to give a sympathetic reading of Anselm's strategy in the Proslogion. We shall extend and refine that reading in what follows.

Why Anselm's Argument for Premise 1 is Invalid

If we take the theory of abstract objects as our starting point, then Anselm's argument for Premise 1 is invalid. Anselm argues that there is something such that nothing greater can be conceived by appealing to the fact that we understand the definite description 'that than which nothing greater can be conceived'. Anselm is implicitly relying here on the suggestion that the description in Premise 1 has an intension, namely, an intentional object existing in the understanding that serves as the content of the description. We actually agree with him on this point, and we also agree with his further supposition that this intentional object *is* such that nothing greater can be conceived. However, we wouldn't agree that the *is* of predication here is to be understood as the *exemplification* form of predication. In the context of the theory of abstract objects, the intension of the description 'the ϕ' is identified as an intentional, abstract object that *encodes* the properties implied by being ϕ. Encoding is a mode of predication distinct from exemplification, though they can both serve as readings for the predicative copula 'is'. Thus, all one can conclude about the description 'that than which nothing greater can be conceived' is that its intension is an object that encodes the properties implied by being such that nothing greater can be conceived. Thus, Anselm cannot validly conclude that the intentional object in the understanding is one that exemplifies the property in question.

Unfortunately, the fact that Anselm's argument for Premise 1 is invalid casts doubt on the truth of Premise 1 and therefore on the soundness of the ontological argument as a whole. As yet, we have no reason to believe that the mere understanding of a definite description entails that there is an object that exemplifies the property of being such that nothing greater can be conceived. Nor do we know of any other argument that can establish that there is such an object. At present, therefore, we have serious doubts whether the ontological argument that we extracted from Proslogion II is sound.

A Sound Argument in Anselm's text

However, we believe a sound argument can be reconstructed from Anselm's text. But as we shall see, its conclusion is not as significant as Anselm wishes it to be.

To reconstruct the sound argument, let's look at Anselm's first premise:

> Premise 1: There is a conceivable thing such that nothing greater can be conceived.

The reading we gave in Oppenheimer and Zalta 1991 (518) of this premise preserved the structure of Anselm's claim, as modern logicians would reconstruct it. Let ϕ_1 be the formula $Cy \,\&\, \neg\exists z(Gzy \,\&\, Cz)$, i.e., y is conceivable and such that no thing z greater than y is conceivable. Now it follows from our argument in the previous section that our original reading of Premise 1 as $\exists y\phi_1$ can't be asserted without further justification. However, there is a true reading of Anselm's claim which can be justifiably asserted and which is true according to object theory, namely, that there is in the understanding an (abstract) object that encodes all and only the properties implied by the property of being a conceivable thing such that nothing greater can be conceived. To see this, note first that from ϕ_1 we may formulate the property $[\lambda y\, \phi_1]$, i.e., $[\lambda y\, Cy \,\&\, \neg\exists z(Gzy \,\&\, Cz)]$. This λ-expression is well-defined in object theory. Now, in terms of this property, the following is an axiom of object theory:

> Premise 1*: $\exists x(A!x \,\&\, \forall F(xF \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz)))$

This asserts that there is an abstract object that encodes all and only the properties implied by the property of being a conceivable thing such that nothing greater can be conceived. So if we read $\exists x$ as "there is in the understanding an x such that", then Premise 1* is our alternative reading of Anselm's Premise 1: it says that there is in the understanding an abstract x that encodes all and only the properties implied by the property $[\lambda y\, \phi_1]$.

We now examine how to reconstruct the rest of Anselm's argument so that we get a valid and sound argument based on Premise 1* for a conclusion that would be reasonably stated in natural language as "God exists". To appreciate the reasoning in what follows, let us move to a new level of generality. Let ψ_1 designate the formula:

$$A!x \,\&\, \forall F(xF \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz)).$$

Thus, Premise 1* may be written as $\exists x\psi_1$. If we now use the abbreviation $\exists!x\chi$ to assert that there is a unique object x such that χ, then the following is a theorem of object theory that will play a role in our new interpretation of the ontological argument

> Lemma 1: $\exists x\psi_1 \rightarrow \exists!x\psi_1$

This asserts that if there is something that encodes all and only the properties implied by the property being such that nothing greater can be conceived, then there is a unique such thing. Lemma 1 is a consequence of the comprehension principle for abstract objects and the following identity principle for abstract objects: $(A!x \,\&\, A!y) \rightarrow (x = y \equiv \forall F(xF \equiv yF))$. For the proof of Lemma 1, assume the antecedent. So there is some abstract object, say b, which encodes exactly the properties implied by $[\lambda y\, \phi_1]$. But, clearly, there couldn't be two such objects, because the identity principle requires that two distinct abstract objects differ by at least one encoded property.

Another important Lemma that will play a role in our new interpretation is the claim that if there is a unique object that encodes exactly the properties implied by $[\lambda y\, \phi_1]$, then that object (i.e., $\imath x\psi_1$) encodes a property F iff F is implied by the property $[\lambda y\, \phi_1]$. In other words, the following is also a theorem of the theory of abstract objects:

> Lemma 2: $\exists!x\psi_1 \rightarrow \forall F(\imath x\psi_1 F \ \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz))$

In the proof of Lemma 2, which appears in the Appendix, we appeal to the Description Axiom that was formulated in our original paper (1991, 513). We will appeal to Lemma 2 to justify an inference in our new interpretation of the ontological argument below.

Next we turn to Anselm's second premise:

> Premise 2: If x is a conceivable thing, then if x doesn't exist, then something greater than x can be conceived.

Notice that we have generalized the version of Premise 2 that appeared in our original paper (1991, 520). Formally, we may represent this premise:

$$\forall x(Cx \rightarrow (\neg E!x \rightarrow \exists z(Gzx \,\&\, Cz)))$$

For purposes of the ontological argument we give below, however, it will be useful to present this premise in one of its equivalent forms. We reach the most useful equivalent form as follows. First, we substitute the contrapositive of the consequent in the universal generalization, yielding:

$$\forall x(Cx \rightarrow (\neg\exists z(Gzx \,\&\, Cz) \rightarrow E!x))$$

Note that this is, in turn equivalent to:

$$\forall x((Cx \,\&\, \neg\exists z(Gzx \,\&\, Cz)) \rightarrow E!x)$$

Finally, if we take advantage of the λ-expression which represents the complex property *being a conceivable thing such that nothing greater can be conceived*, then the previous claim is equivalent to:

$$\forall x([\lambda y\, Cy \,\&\, \neg\exists z(Gzy \,\&\, Cz)]x \rightarrow E!x)$$

In what follows, then, we use our abbreviation ϕ_1 to state this premise as:

Premise 2*: $\forall x([\lambda y\, \phi_1]x \rightarrow E!x)$

That is, the property of being such that nothing greater can be conceived implies the concept of existence.

We are now ready to assemble these pieces (Premise 1*, Premise 2*, and the above abstraction principle) into a new reading of Anselm's argument which shows it to be both valid and sound:

1.	$\exists x\psi_1$	Premise 1*
2.	$\forall z([\lambda y\, \phi_1]z \rightarrow E!z)$	Premise 2*
3.	$\exists!x\psi_1$	From (1), by Lemma 1
4.	$\forall F(\imath x\psi_1 F \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz))$	From (3), by Lemma 2
5.	$\imath x\psi_1 E! \equiv \forall z([\lambda y\, \phi_1]z \rightarrow E!z)$	From (4), by UE
6.	$\imath x\psi_1 E!$	From (5) and (2), by $\equiv$E
7.	$g = \imath x\psi_1$	Definition of 'g'
8.	$gE!$	From (6) and (7), by =E

Now in object theory, encoding predication is a genuine mode of predication and is therefore regarded as a way to disambiguate ordinary predication. Thus, conclusion (8) is a reading of the ordinary-language claim "God exists".

So, on this alternative reading of the argument, Anselm argues not only validly but also soundly! However, his conclusion is not as strong as he would like it to be. He reaches the conclusion "God exists", but only in its reading as the encoding claim $gE!$, not in its reading as the exemplification claim $E!g$.

Appendix

Proof of Lemma 2. Assume $\exists!x\psi_1$. I.e., assume

$$\exists!x(A!x \,\&\, \forall F(xF \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz)))$$

Now we want to show:

$$\forall F(\imath x\psi_1 F \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz))$$

So we pick an arbitrary property, say, P, and show:

$$\imath x\psi_1 P \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Pz)$$

($\rightarrow$) So assume $\imath x\psi_1 P$. Note that this is an atomic encoding formula. We may then appeal to the Description Axiom (1991, 513), which asserts, for any atomic or identity formula $\chi(z)$ in which a description of the form $\imath x\psi$ has been substituted for z:

$$\chi^{\imath x\psi}_z \equiv \exists x(\psi \,\&\, \forall y(\psi^y_x \rightarrow y\!=\!x) \,\&\, \chi^x_z)$$

So let $\chi(z)$ be the formula zP. Since we have assumed $\imath x\psi_1 P$ (i.e., $\chi^{\imath x\psi_1}_z$), the above Description Axiom allows us to conclude, for some object, say d, that

$$\psi_1{}^d_x \,\&\, \forall y(\psi_1{}^y_x \rightarrow y\!=\!x) \,\&\, dP$$

In other words, expanding ψ_1:

$$\begin{aligned}&A!d \,\&\, \forall F(dF \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz)) \,\&\\ &\forall y(A!y \,\&\, \forall F(yF \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz)) \rightarrow y\!=\!d) \,\&\\ &dP\end{aligned}$$

But the second conjunct of this fact about d is:

$$\forall F(dF \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz))$$

and so in particular, $dP \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Pz)$. The final conjunct of our fact about d is dP. So it follows that $\forall z([\lambda y\, \phi_1]z \rightarrow Pz)$.

($\leftarrow$) Now assume $\forall z([\lambda y\, \phi_1]z \rightarrow Pz)$. We want to show $\imath x\psi_1 P$. We know, by the first assumption in our proof, that $\exists!x\psi_1$. In other words, we know, for some object, say k, that:

$$\begin{aligned}&A!k \,\&\, \forall F(kF \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz)) \,\&\\ &\forall y(A!y \,\&\, \forall F(yF \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz)) \rightarrow y\!=\!k)\end{aligned}$$

But from this we have $\forall F(kF \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz))$, and it therefore follows that $kP \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Pz)$. Since our hypothesis in the $(\leftarrow)$ direction is that $\forall z([\lambda y\, \phi_1]z \rightarrow Pz)$, it follows that kP. So we now know:

$$\begin{array}{l} A!k \;\&\; \forall F(kF \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz)) \;\&\; \\ \forall y(A!y \;\&\; \forall F(yF \equiv \forall z([\lambda y\, \phi_1]z \rightarrow Fz)) \rightarrow y{=}k) \;\&\; \\ kP \end{array}$$

So by the Description Axiom, $\imath x\psi_1 P$. $\bowtie$

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