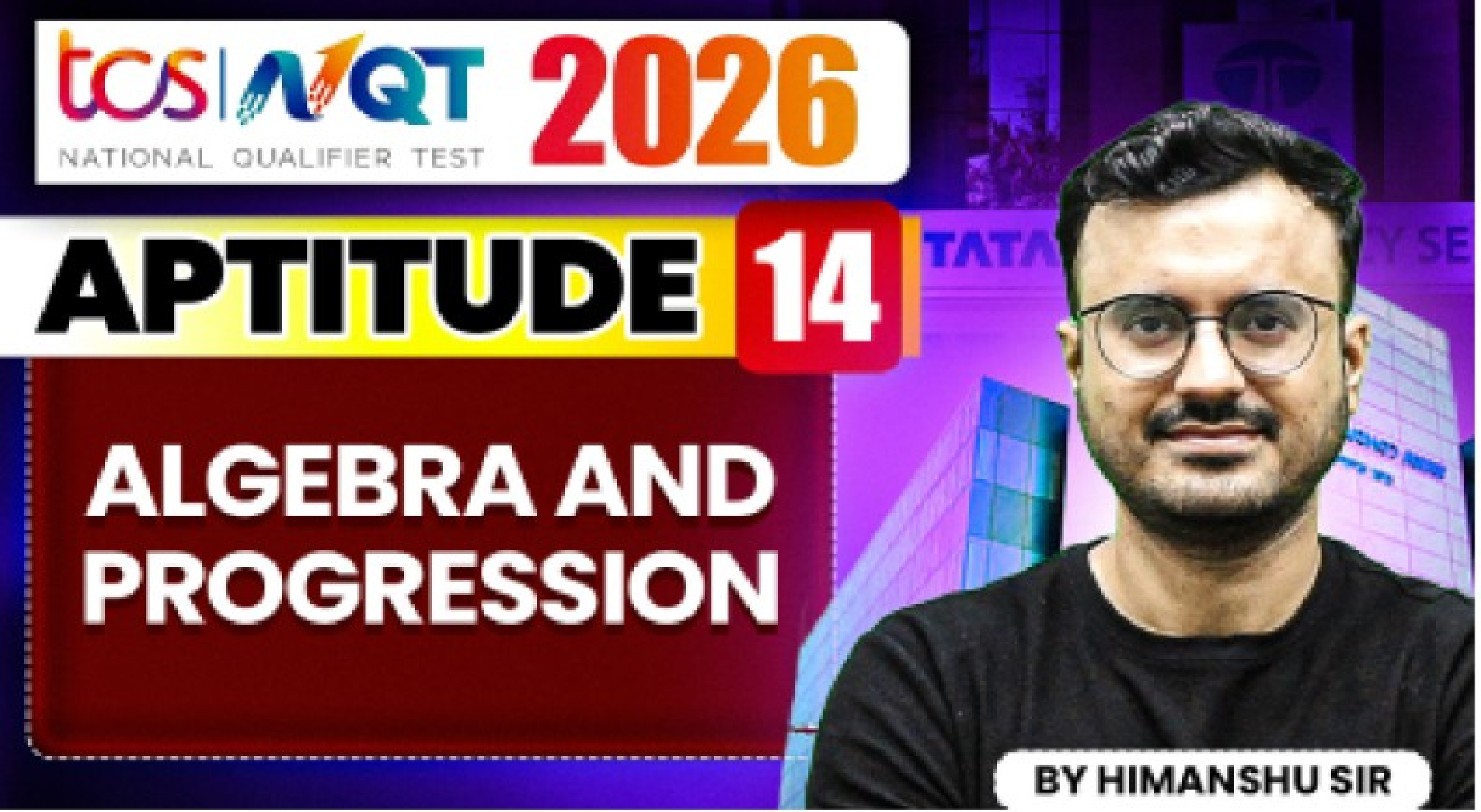

tcs NQT
NATIONAL QUALIFIER TEST
2026
APTITUDE
14
ALGEBRA AND PROGRESSION
BY HIMANSHU SIR

ALGEBRA & PROGRESSION

Progression:→

1.) A·P [Arithmatic progression]:→

$$\overset{1^{st}}{a},\ \overset{2^{nd}}{a+1d},\ \overset{3^{rd}}{a+2d},\ a+3d,\ a+4d\ ----,\ \overset{n^{th}\text{ term}}{a+(n-1)d}$$

+d +d +d

a → first term

d → common diff.

n → no. of terms

✓ n^{th} term → $a_n = a+(n-1)d$

✓ $S_n \Rightarrow \frac{n}{2}\left[2a+(n-1)d\right]$ - or - $S_n = \frac{n}{2}\left[\text{first term} + \text{Last term}\right]$

- or -

$S_n = n \times$ Avg. of terms

No. of terms in A·P

Sum of three terms

1) 3 terms in A·P $\Rightarrow$ $\underbrace{a-d, \; a}_{+d}, \; \underbrace{\; a+d}_{+d}$ $\Rightarrow$ $a-\cancel{d}+a+a+\cancel{d} \Rightarrow 3a$

2) 5 terms in A·P $\Rightarrow$ $a-2d, a-d, a, a+d, a+2d \Rightarrow$ Sum of A·P $5a$ ✓

3) 4 terms in A·P $\Rightarrow$ $\underbrace{a-3d, \; a-d}_{+2d}, \; \underbrace{a-d, \; a+d}_{+2d}, \; \underbrace{a+d, \; a+3d}_{+2d}$ $\Rightarrow$ Sum of A·P $4a$ ✓

A·M [Arithmatic Mean]:→

If a, b are in A·P, now insert a term x b/n a and b such that a, x, b also in A·P.

a, x, b also in A·P

$\Downarrow$

$\Rightarrow (x-a) = (b-x)$

$2x = a+b$

$\boxed{A \cdot M}$ $\boxed{x = \frac{a+b}{2}}$

2.) G. P. [Geometric progression] :→

$\underline{ar^0}$ $\quad a, \overset{2nd}{ar^{1}}, \overset{3rd}{ar^2}, ar^3, \ldots\ldots, ar^{n-1}$

$\times r \quad \times r \quad \times r$

ar^{n-1} ← n^{th} term

$a \rightarrow$ first term

$r \rightarrow$ common ratio

$n \rightarrow$ no. of terms

n^{th} term $\Rightarrow a \times r^{n-1}$

Sum of G.P $\Rightarrow$ if $\boxed{r>1}$

$$S_n = \frac{a[r^n - 1]}{[r-1]}$$

if $0<r<1$

$$S_n = \frac{a[1-r^n]}{(1-r)}$$

** Sum of Infinite G.P

Case-I if $r > 1$

$$S_\infty = \infty$$

2^∞

$(0.5)^1 = 0.5$

$(0.5)^2 = 0.25$

$(0.5)^3 = 0.125$

Case-II if $0 < r < 1$

$$S_n = \frac{a\left[1 - \cancel{r^n}^{\,0}\right]}{[1-r]}$$

$r \to 0 < r < 1$

$$S_\infty = \frac{a}{1-r}$$

$$S_n = \frac{a(1)}{1-r}$$

$r^\infty \Rightarrow [0 \text{ to } 1]^\infty$

$\Rightarrow 0$

G.M [Geometric mean]

if a and b are in G.P, then insert a term x b/w a and b such that a, x, b also in G.P.

a, x, b are in G.P

$\Downarrow$

$$\frac{x}{a} = \frac{b}{x}$$

$$x^2 = ab$$

GM

$$\boxed{x = \sqrt{ab}}$$

8) H.P [Harmonic progression] :→

if a, b, c, d are in H.P

then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ are in A.P

ex) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$

are in H.P

(2, 4, 6, 8) ← are in A.P

Harmonic Mean [H.M] :→

If a and b are in H.P, then insert a term x b/n a and b such that a, x, b also in H.P.

→ a, x, b in H.P

⇓

$\frac{1}{a}, \frac{1}{x}, \frac{1}{b}$ are in A.P

$$\frac{1}{x} - \frac{1}{a} = \frac{1}{b} - \frac{1}{x}$$

$$\frac{2}{x} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{2}{x} = \frac{a+b}{ab}$$

$$\boxed{x = \frac{2ab}{a+b}}$$

H.M

Relation btn AM, GM and H.M

1.) $\boxed{AM \times H.M = GM^2}$

$\Rightarrow \frac{\cancel{a+b}}{\cancel{2}} \times \frac{\cancel{2}ab}{\cancel{a+b}}$

$\Rightarrow ab = (\sqrt{ab})^2 \Rightarrow (GM)^2$

2.) $\boxed{AM \geq GM \geq HM}$

Q. If $x + \frac{1}{x} = 5$, then find the value of $x^2 + \frac{1}{x^2}$.

~~A) 23~~
B) 25
C) 27
D) 20

TCS NQT PYQ

$$x^2 + \frac{1}{x^2} = 5^2 - 2$$
$$= 23$$

Q. Solve for x: $3^{(x-1)} + 3^{(x+1)} = 90$.

A) 2
~~B) 3~~
C) 4
D) 5

$$3^{(x-1)} + 3^{(x+1)} = 90$$

find value of x?

$$3^{x-1} + \underline{3^{x-1} \times 3^2} = 90$$

$$3^{x-1}\left[1 + 3^2\right] = 90$$

$$3^{x-1} \times \cancel{10} = \cancel{90}^{\,9}$$

$$3^{x-1} = 3^2$$

$x - 1 = 2$

$x = 3$ ✓

Q. Find the 15th term of the Arithmetic Progression: 7, 13, 19, 25, ...

A) 85

B) 91

C) 97

D) 103

TCS NQT PYQ

$d = 6$

$a_{15} = \underline{a + 14d}$

$\Rightarrow 7 + 14 \times 6$

$\Rightarrow 7 + 84$

$\Rightarrow 91$

Q. If the sum of the first 10 terms of an AP is 155 and the common difference is 3 , find the first term.

A) 1
~~B) 2~~
C) 4
D) 5

TCS NQT PYQ

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$\cancel{155}^{31} = \frac{\cancel{10}^{\cancel{5}^{1}}}{\cancel{2}}\left[2a+9\times 3\right]$$

$$31 = 2a+27$$

$$4 = 2a$$

$$a=2$$

Q. The 3rd and 6th terms of a Geometric Progression are 12 and 96 respectively. Find the 1st term.

A) 2

B) 3

C) 4

D) 6

TCS NOT PYQ

$a_3 = a \times r^2 = 12 \quad \text{— (i)}$

$a_6 = a \times r^5 = 96 \quad \text{— (ii)}$

(ii)/(i)

$$\frac{\cancel{a} \times r^5}{\cancel{a} \times r^2} = \frac{\cancel{96}^{\,8}}{\cancel{12}}$$

$r^3 = 8$

$r = 2$

$a \times 2^2 = 12$

$a = 3$

Q. What is the sum of the infinite GP series: 18,6,2,$\frac{2}{3}$, ... ?

[TCS NQT PYQ]

A) 24
~~B)~~ 27
C) 30
D) 36

$18, 6, 2, \frac{2}{3}, \ldots\ldots \infty$

$\underbrace{18, 6}_{\div 1/3}, \underbrace{6, 2}_{\div 1/3}$

$r = 1/3 \rightarrow 0 < r < 1$

$a_\infty = \frac{a}{1-r} = \frac{18}{1-1/3} = \frac{18}{2/3} = \frac{\cancel{18}^{9} \times 3}{\cancel{2}} = 27$

Q. Find the Arithmetic Mean (AM) and Geometric Mean (GM) of 4 and 16.

TCS NQT PYQ

A) $\text{AM} = 10, \text{GM} = 8$

B) $\text{AM} = 8, \text{GM} = 10$

C) $\text{AM} = 12, \text{GM} = 8$

D) $\text{AM} = 10, \text{GM} = 64$

$AM = \frac{4+16}{2} \Rightarrow 10$

$GM = \sqrt{4 \times 16} \Rightarrow 8$

Q) Sum of $4 + 44 + 444 + \cdots$ n terms

(A) $\frac{40}{81}[8^n - 1] - \frac{5n}{9}$

(B) $\frac{40}{81}(8^n - 1) - \frac{4n}{9}$

(C) $\frac{40}{81}(10^n - 1) - \frac{4n}{9}$

(D) $\frac{40}{81}(10^n - 1) - \frac{5n}{9}$

Sol^n)

$4[1 + 11 + 111 + \cdots \text{ n terms}]$

$\frac{4}{9}[9 + 99 + 999 + \cdots \text{ n terms}]$

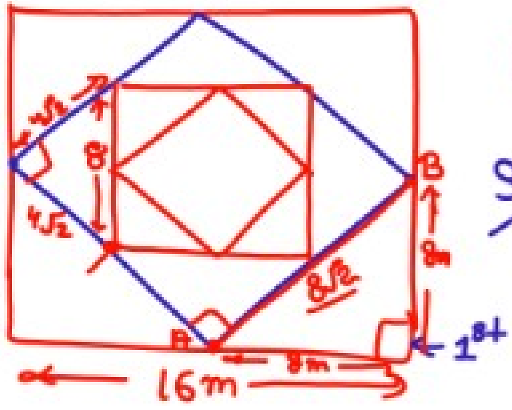
$\frac{4}{9}[(10-1) + (100-1) + (1000-1) + \cdots \text{ n terms}]$

$\frac{4}{9}[(\underbrace{10 + 100 + 1000 + \cdots \text{ n terms}}_{G.P}) - (1 + 1 + \cdots \text{ n terms})]$

$\frac{4}{9}\left[10 \times \frac{(10^n - 1)}{10 - 1} - n\right]$

$\frac{40}{81}(10^n - 1) - \frac{4n}{9}$

Q)



Find sum of area of all squares?

Sol)

$A_1 = 16 \times 16 = 256\ m^2$

$A_2 = 8\sqrt{2} \times 8\sqrt{2} = 128\ m^2$

$A_3 = 8 \times 8 = 64$

$A_1 + A_2 + A_3 + \cdots \infty$ ∴ Infinite G.P

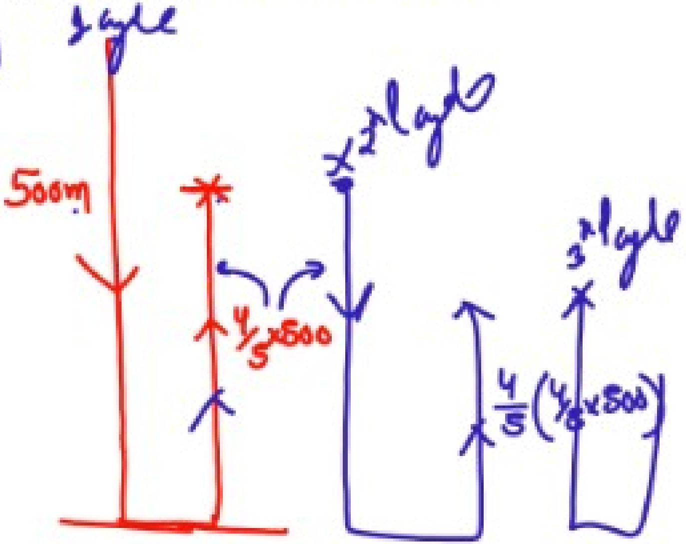
$256 + 128 + 64 + \cdots \infty$

$r = 1/2$

$S_\infty = \frac{256}{1-1/2} \Rightarrow \frac{256}{1/2} = 512\ m^2$

Q) If a ball is thrown from a height of 500 m, if ball jumps $\frac{4}{5}$th of its last jump each time. Find the total distance covered by ball till it comes to rest.

Sol^n)



$$D_1 + D_2 + D_3 - - - - - \infty$$

$$\left(500+\frac{4}{\cancel{5}}\times \overset{100}{\cancel{500}}\right)+\left(\frac{4}{5}\times 500+\frac{4}{5}\times\frac{4}{5}\times 500\right)+ - - - -$$

$$900 + \frac{4}{5}\times 900 + - - - - - - \infty$$

$$r = 4/5$$

$$S_\infty = \frac{900}{1-4/5} = \frac{900}{1/5} \Rightarrow 4500m$$

ALGEBRA

Q. If $x^4 + y^4 = 19$ and $x + y = 1$, Find $x^2y^2 - 2xy = ?$.

(a) 10

(b) 9

(c) 8

(d) 7

$x^2y^2 - 2xy = \frac{18}{2} = 9$

$(x+y)^2 = 1^2$

$x^2+y^2+2xy = 1$

$x^2+y^2 = 1-2xy$

again square both side

$(x^2+y^2)^2 = (1-2xy)^2$

$x^4+y^4+2x^2y^2 = 1+4x^2y^2-4xy$

$19-1 = 2x^2y^2-4xy$

$18 = 2x^2y^2-4xy$

Q. Factor of $x^{29} - x^{26} - x^{23} + 1$.

(a) (x-1) but not (x+1)

(b) (x+1) but not (x-1)

(c) both (x+1) and (x-1)

(d) Neither (x+1) nor (x-1)

$$\boxed{f(x) = x^{29} - x^{26} - x^{23} + 1}$$

* Remainder Theorem

$(x-1) \Rightarrow$ put $x - 1 = 0 \rightarrow x = 1$

$f(x=1) \Rightarrow (1)^{29} - (1)^{26} - 1^{23} + 1$

$1 - 1 - 1 + 1 = 0$

$R = 0$

for $x+1 \Rightarrow 0 \rightarrow x = -1$

$f(x=-1) \Rightarrow (-1)^{29} - (-1)^{26} - (-1)^{23} + 1$

$\Rightarrow -1 - 1 + 1 + 1$

$\Rightarrow 0$

$(x+1)$ and $(x-1)$ are factor of $(x^{29} - x^{26} - x^{23} + 1)$

$(-1)^{even} = +1$

$(-1)^{odd} = -1$

Q. If (x+1) and (x-1) are factors of $ax^3 + bx^2 + 3x + 5$, find the value of a and b ?

(a) 3, 5

(b) -3, -5

(c) -3, 4

(d) -4, 5

if $x+1$ is factor, put $x+1=0$ $(x=-1)$ — final R=0

$$f(x=-1) = a\times(-1)^3 + b(-1)^2 + 3(-1) + 5 = 0$$

$$-a+b-3+5=0$$

$$-a+b+2=0$$

$$-a+b=-2$$

$$\boxed{a-b=2}$$

(x-1) is also factor → put $x-1=0$ $(x=1)$

$$f(x=1) \Rightarrow a\times1^3 + b\times1^2 + 3\times1 + 5 = 0$$

$$\boxed{a+b=-8}$$

$$a+b=-8$$
$$a-b=2$$

$$a = \frac{-8+2}{2} \Rightarrow -3$$

$$b = \frac{-8-2}{2} = -5$$

Q. $x^2 + bx + 7$ is divided by $(x-1)$ leaves remainder = 12, find b?.

(a) 1

(b) 2

(c) 3

(d) 4

$x - 1 = 0 \Rightarrow x = 1$

$f(x=1) \Rightarrow 12$

$1^2 + b \times 1 + 7 = 12$

$1 + b + 7 = 12$

$b = 4$

Q. find the H.C.F of the polynomial $30(x^2 - 3x+2)$ and $50(x^2 - 2x + 1)$.

(a) 10

(b) $30(x-1)$

(c) $10(x-1)^2$

(d) None

H.C.F → have to select least power

$f(x) \Rightarrow 30(x^2-3x+2)$

$\Rightarrow 2\times3\times5\times(x-1)(x-2)$

$g(x) = 50\times(x^2-2x+1)$

$\Rightarrow 2\times5^2\times(x-1)^2$

$H.C.F \Rightarrow 2\times5\times(x-1)$

$\Rightarrow 10(x-1)$

Q. find H.C.F of $\overset{f(x)}{30x^3} - 30x^2 - 15x + 27$ and

$g(x)$ $30x^3 - 61x^2 - 24x + 10$.

(a) ~~$31x^2 + 29x + 17$~~

(b) $31x^2 + 9x + 17$

(c) ~~$31x^2 - 9x - 27$~~

(d) ~~$30x^2 + 11x + 11$~~

Long divisn method

H.C.F

72, 90

Diff ⇒ 18 ← H.C.F will 18 or its factor

$f(x) - g(x) = \cancel{30x^3} - 30x^2 - 15x + 27 - \cancel{30x^3} + 61x^2 + 24x - 10$

$\Rightarrow (31x^2 + 9x + 17)$

↑ H.C.F or its factor

Note)

$$x^n + \frac{1}{x^n} = K$$

then $x^{2n} + \frac{1}{x^{2n}} = K^2 - 2$

Sol)

$$\left(x^n + \frac{1}{x^n}\right)^2 = (K)^2$$

$$x^{2n} + \frac{1}{x^{2n}} + 2 \times \cancel{x^n} \times \frac{1}{\cancel{x^n}} = K^2$$

$$x^{2n} + \frac{1}{x^{2n}} = K^2 - 2$$

Note)

if $x^n - \frac{1}{x^n} = K$

$$x^{2n} + \frac{1}{x^{2n}} = K^2 + 2$$

Sol)

$$\left(x^n - \frac{1}{x^n}\right)^2 = (K)^2$$

$$x^{2n} + \frac{1}{x^{2n}} - 2 \times \cancel{x^n} \times \frac{1}{\cancel{x^n}} = K^2$$

$$x^{2n} + \frac{1}{x^{2n}} = K^2 + 2$$

Q. $\sqrt{x}+\frac{1}{\sqrt{x}}=1$, find $x^{512}+\frac{1}{x^{512}}=?$

(a) 1

~~(b) -1~~

(c) 512

(d) 256

Solⁿ

$x^{512}+\frac{1}{x^{512}}=-1$

$\sqrt{x}+\frac{1}{\sqrt{x}}=1$

square both side

$x+\frac{1}{x}=1^2-2=-1$

$x^2+\frac{1}{x^2}=(-1)^2-2=-1$

$x^4+\frac{1}{x^4}=-1$

soon

Q. If $x^2 + x + 1 = 0$

Find $x^3 + 1 = ?$

(a) 2

(b) -1

(c) 0

(d) 4

$x^3 + 1$

$1 + 1$

2

$x^2 + x + 1 = 0$

$x^2 + 1 = -x \Rightarrow x^2 + x = -1$

Now divide by x both side

$$\boxed{x + \frac{1}{x} = -1}$$

$x^2 + x + 1 = 0$

$x^3 + \underline{x^2 + x} = 0$

$x^3 + -1 = 0$

$x^3 = 1$

if $x + \frac{1}{x} = -1$

$x^3 = 1$

if $x + \frac{1}{x} = 1$

then $x^3 = -1$

Q. If $\sqrt{x}+\frac{1}{\sqrt{x}}=1$, find $x^{40}+\frac{1}{x^{40}}=?$

(a) -1

(b) 1

(c) 0

(d) 40

$\sqrt{x}+\frac{1}{\sqrt{x}}=1$

$x+\frac{1}{x}=-1$ ✓

$\Downarrow$

$\boxed{x^3=1}$ ✓

$x^{40}+\frac{1}{x^{40}}$

$x^{39}\times x+\frac{1}{x^{39}\times x}$

$(x^3)^{13}\times x+\frac{1}{(x^3)^{13}\times x}$

$x+\frac{1}{x}$

$\Downarrow$

-1 Ans

Q. If $x^2 + x = 5$

$$(x+3)^3 + \frac{1}{(x+3)^3} = ?$$

(a) 110

(b) 40

(c) 85

(d) 120

Try Yourself

Q. If $x^4 + \frac{1}{x^4} = 23$

Find (i) $x^2 + \frac{1}{x^2} = ?$

(ii) $x - \frac{1}{x} = ?$

(iii) $x + \frac{1}{x} = ?$

$\boxed{x^3 - 1 = 0}$

cubic eqn

$x^3 = 1$

$x = 1$ or $x = k_1$, $x = k_3$

i) $x^4 + \frac{1}{x^4} + 2 = 23 + 2$

$\rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 25 \Rightarrow x^2 + \frac{1}{x^2} = 5$ ✓

ii) $x^2 + \frac{1}{x^2} = 5$

$x^2 + \frac{1}{x^2} - 2 = 5 - 2$

$\left(x - \frac{1}{x}\right)^2 = 3 \Rightarrow x - \frac{1}{x} = \sqrt{3}$ ✓

(iii) $x^2 + \frac{1}{x^2} + 2 = 5 + 2$

$x + \frac{1}{x} = \sqrt{7}$ Ans

Q. If $x + \frac{1}{x} = 3$, find $x^7 + \frac{1}{x^7} = ?$

(a) 567

(b) 945

(c) 480

(d) None

$\left(x^3 + \frac{1}{x^3}\right)\left(x^4 + \frac{1}{x^4}\right)$

Try Yourself

$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$x + \frac{1}{x} = 3$

$x^2 + 1/x^2$

$x^4 + 1/x^4 \rightarrow$

$$\boxed{\begin{aligned} x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right)^3 - 3\left(x + 1/x\right) \\ x^3 - \frac{1}{x^3} &= \left(x - \frac{1}{x}\right)^3 + 3\left(x - 1/x\right) \end{aligned}}$$

$a^3 - b^3 = (a-b)^3 - 3ab(a-b)$