

Package ‘kza’

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Title Kolmogorov-Zurbenko Adaptive Filters

Description Time series analysis including break detection, spectral analysis, and Kolmogorov-Zurbenko Fourier transforms, following Zurbenko et al. (1996) <doi:10.1175/1520-0442(1996)009%3C3548:DDITSO%3E2.0.CO;2> and Yang and Zurbenko (2010) <doi:10.1002/wics.71>.

Suggests polynom, testthat

SystemRequirements fftw (>= 3.2.2)

License GPL-3

NeedsCompilation yes

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kz	<i>Kolmogorov-Zurbenko filter</i>
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Description

Kolmogorov-Zurbenko low-pass linear filter.

Usage

```
kz(x, m, k = 3)
```

Arguments

x	The raw data that will be smoothed. The data can have as many as 3 dimensions. KZ will also handle a time series.
m	Window size for the filter. This can be up to 3 dimensions, but not more than the dimensionality of the input data x.
k	Number of iterations.

Details

KZ is an iterated moving average. The filter can be used with missing values. One iteration is equivalent to a simple moving average. Three iterations is an approximately Gaussian shaped filter.

References

Zurbenko, I. G., 1986: The spectral Analysis of Time Series. North-Holland, 248 pp.

Examples

```
## 2 dimensions
set.seed(2)
a <- matrix(rep(0,100*100),nrow=100)
a[35:70,35:70]<-1
a <- a + matrix(rnorm(100*100,0,1),nrow=100)

z<-kz(a,m=c(20,5),k=3)
x <- seq(1,100)
y <- x
op <- par(bg = "white")

c="lightblue"
m="Unsmoothed"
persp(x, y, a, zlab="a", ticktype="detailed", theta = 60, phi = 45, col = c, main=m)

m="KZ(a,m=c(20,5),k=3)"
persp(x, y, z, zlab="z", ticktype="detailed", theta = 60, phi = 45, col = c, main=m)
```

```
#example
t <- seq(0,20,length=20*365)
set.seed(6); e <- rnorm(n = length(t), sd = 2.0)
y <- sin(3*pi*t) + e
z <- kz(y,30)

par(mfrow=c(2,1))
plot(y,ylim=c(-5,5),type="l",main="y = sin(3*pi*t) + noise")
plot(z,ylim=c(-5,5), type="l",main="KZ filter")
lines(sin(3*pi*t), col="blue")
par(mfrow=c(1,1))
```

kza

Kolmogorov-Zurbenko Adaptive

Description

KZA will recover 2-dimensional or 3-dimensional image or signal buried in noise.

Usage

```
kza(x, m, y = NULL, k = 3, min_size = round(0.05*m), tol = 1.0e-5, impute_tails = FALSE,
    symmetrize = FALSE, normalize = c("max", "quantile"))
## S3 method for class 'kza'
plot(x, ...)
```

Arguments

<code>x</code>	A vector of the time series or a matrix (2d) or an array (3d) of an image.
<code>m</code>	The full window width, on the same scale as <code>kz()</code> : the per-side radius used internally is <code>floor(m/2)</code> , and the same radius drives the default <code>kz</code> baseline. (Before 4.2.0, <code>kza</code> used <code>m</code> itself as the per-side radius, so its adaptive window was roughly twice the baseline's for the same <code>m</code> ; double <code>m</code> to reproduce old results.)
<code>y</code>	The filtered output from <code>kz</code> .
<code>k</code>	The number of iterations.
<code>min_size</code>	Minimum size of window <code>q</code> .
<code>tol</code>	The smallest value to accept as nonzero.
<code>impute_tails</code>	The default is to drop the tails.
<code>symmetrize</code>	Matrix input only: average the filter over the four 90-degree rotations of the input. The adaptive head/tail rule can misallocate the window immediately beside a sharp, high-contrast, axis-aligned edge, so plain <code>kza</code> does not commute with 90-degree rotation; rotation averaging cancels that row/column bias (exactly, for square input) at four times the compute. Default FALSE.

normalize	Normalizer for the adaptive shrink factor. "max" (the default, and the published behavior) uses the literal maximum of the difference metric; "quantile" uses its 99th percentile, so a handful of extreme pixels do not single-handedly set the smoothing scale for the whole image, with an automatic fallback to the maximum whenever the quantile is zero (sparse structure on an otherwise flat field would otherwise be smoothed away).
...	Other parameters.

Details

The selection of parameters of KZA depend on the nature of the data. This function may take a long time to run, depending on the number of dimensions and the size of the dimensions.

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References

I. Zurbenko, P.S. Porter, S.T. Rao, J.Y. Ku, R. Gui, R.E. Eskridge Detecting Discontinuities in Time Series of Upper-air Data: Development and Demonstration of an Adaptive Filter Technique. Journal of Climate: (1996) Vol. 9, No. 12, pp. 3548-3560. doi:[10.1175/15200442\(1996\)009<3548:DDITSO>2.0.CO;2](https://doi.org/10.1175/15200442(1996)009<3548:DDITSO>2.0.CO;2)

Kevin L. Civerolo, Elvira Brankov, S. T. Rao, Igor Zurbenko Assessing the impact of the acid deposition control program. Atmospheric Environment 35 (2001) 4135-4148 <http://www.elsevier.com/locate/atmosenv>

J.Chen, I.Zurbenko, Nonparametric Boundary detection, Communications in Statistics, Theory and Methods, Vol.26, 12, 2999-3014, 1997.

Examples

```
#####
# this is an example of detection of a break point in a time series
#####
yrs <- 20
t <- seq(0,yrs,length=yrs*365)
m <- 365

#noise
e <- rnorm(n = length(t),0,1)
trend <- seq(0,-1,length=length(t))

#signal
bkpt <- 3452
brk <- c(rep(0,bkpt),rep(0.5,length(t)-bkpt))
signal <- trend + brk

# y = seasonal + trend + break point + noise
y <- sin(2*pi*t) + signal + e

k.kz <- kz(y,m)
```

```

# kza reconstruction of the signal
k.kza <- kza(y,m,y=k.kz,min_size=10)

par(mfrow=c(2,1))
plot(y,type="l", ylim=c(-3,3))
plot(signal,type="l",ylim=c(-3,3),
      main="Signal and KZA Reconstruction")
lines(k.kza$kza, col=4)

#####
# image detection (2d)
#####
set.seed(2)
a <- matrix(rep(0,100*100),nrow=100)
a[35:70,35:70]<-1
a <- a + matrix(rnorm(100*100,0,1),nrow=100)
y<-kz(a,m=15,k=3)
v <- kza(a,m=15,y=y,k=3,impute_tails=TRUE)

x <- seq(1,100)
y <- x
op <- par(bg = "white")

###
#noise
###
c="lightblue"
persp(x, y, a, zlab="z", zlim=c(-5,5), ticktype="detailed", theta=30, phi=30, col=c)

###
#kza filtered
###
persp(x,y,v$kza,zlab="z",zlim=c(-5,5),ticktype="detailed",theta=30,phi=30,col=c)

###
# another view
###
par(mfrow=c(1,2))
image(a,col=gray(seq(0,1,1/255)))
image(v$kza,col=gray(seq(0,1,1/255)))
par(mfrow=c(1,1))

```

Description

Kolmogorov-Zurbenko Fourier Transform is an iterated Fourier transform.

Usage

```
kzft(x, f=0, m=1, k=1)
coeff(m, k)
transfer_function(m, k, lamda = seq(-0.5,0.5,by=0.01), omega = 0 )
```

Arguments

x	The raw data
f	The frequency that KZFT is applied at.
m	The window size for transform. Must be odd: KZFT's band-pass transfer-function properties are proven only for odd window lengths (Valachovic 2025); an even m is rejected rather than silently misaligned.
k	The number of iterations for applying the KZFT
lamda	The frequencies used for the calculating the transfer function.
omega	The frequency that KZFT is applied at.

Details

Kolmogorov-Zurbenko Fourier Transform (KZFT) is the Fourier transform applied over every segment of length m iterated k times. The argument `alg="F"` will use Fast Fourier Transforms written in C (fftw library). The `alg="C"` is a slow Fourier Transform but has the advantage of being able to handle missing values. It currently works in one dimension. The `alg="R"` is an R version of KZFT for experimental purposes. The `coeff` function generates the coefficients for the KZFT function.

You will introduce a phase shift and decrease the fidelity of the signal if the product of $f \cdot m$ is not an integer.

References

- I. G. Zurbenko, The spectral Analysis of Time Series. North-Holland, 1986.
- I. G. Zurbenko, P. S. Porter, Construction of high-resolution wavelets, Signal Processing 65: 315-327, 1998.
- R. Neagu, I. G. Zurbenko, Tracking and separating non-stationary multi-component chirp signals, J. Franklin Inst., 499-520, 2002.
- R. H. Shumway, D. S. Stoffer, Time Series Analysis and Its Applications: With R Examples, Springer, 2006.
- Wei Yang and Igor Zurbenko, kzft: Kolmogorov-Zurbenko Fourier Transform and Applications, R-Project 2007.
- Igor G. Zurbenko, Amy L. Potrzeba, Tidal Waves in Atmosphere and Their Effects, Acta Geophysica Volume 58, Number 2, 356-373
- M. Valachovic, A Kolmogorov-Zurbenko Fourier Transform Band-pass Filter Extension for Time Series Analysis, arXiv:2503.23493, 2025.

See Also

[kzp](#), [kztp](#),

Examples

```
# example taken from Wei Yang's KZFT package
# coefficients of kzft(201,5)

# function to calculate polynomial coefficients for kzft
## Not run:
a<-coeff(201,5);
t<-seq(1:1001)-501;
z<-cos(2*pi*0.025*t);
plot(z*a,type="l",xlab="Time", ylab="Coefficient", main="Coefficients of the kzft");
lines(a);
lines(-1*a);

## End(Not run)

# example taken from Wei Yang's KZFT package
# transfer function of the kzft(201,5) at frequency 0.025
lamda<-seq(-0.1,0.1,by=0.001)
tf1<-transfer_function(201,1,lamda,0.025)
tf2<-transfer_function(201,5,lamda,0.025)
matplot(lamda,cbind(log(tf1),log(tf2)),type="l",ylim=c(-15,0),
  ylab="Natural log transformation of the coefficients",
  xlab="Frequency (cycles/time unit)",
  main="Transfer function of kzft(201,5) at frequency 0.025")

# example with missing values
set.seed(2)
period=101
f<-1/period
t<-1:2000
s<-1*sin(2*pi*f*t)
x<-s
noise<-3*rnorm(length(t))
x<-s+noise
m=101

rand_idx <- sample(t,100,replace=FALSE)
x[rand_idx]<-NA
x<-as.vector(na.omit(x))

system.time(z1<-kzft(x, m=m, k=1, f=f))
system.time(z2<-kzft(x, m=m, k=2, f=f))
system.time(z3<-kzft(x, m=m, k=3, f=f))

par(mfrow=c(2,2))
plot(x,type="l",main="Original time series",xlab="t", ylab="y")
lines(s,col="blue")
plot(2*Re(z1),type="l",main="kzft(101,1)",xlab="t", ylab="y", ylim=c(-6,6))
lines(s,col="blue")
plot(2*Re(z2),type="l",main="kzft(101,2)",xlab="t", ylab="y", ylim=c(-6,6))
lines(s,col="blue")
plot(2*Re(z3),type="l",main="kzft(101,3)",xlab="t", ylab="y", ylim=c(-6,6))
```

```
lines(s,col="blue")
par(mfrow=c(1,1))
```

kzp

Kolmogorov-Zurbenko Periodogram

Description

Kolmogorov-Zurbenko periodogram and smoothing using DiRienzo-Zurbenko (DZ).

Usage

```
kzp(y, m=length(y), k=1)
## S3 method for class 'kzp'
smooth(object, log=TRUE, smooth_level=0.05, method = "DZ")
## S3 method for class 'kzp'
nonlinearity(x)
## S3 method for class 'kzp'
variation(x)
## S3 method for class 'kzp'
summary(object, digits=getOption("digits"), top=1, ...)
## S3 method for class 'kzp'
plot(x, ...)
```

Arguments

y	The raw data.
m	The width of filtering window. Must be odd; KZFT's band-pass transfer-function properties are proven only for odd window lengths.
k	The number of iterations for the KZFT
object	Output from kzp function.
log	Use logarithm values for smoothing.
smooth_level	Percentage of smoothness to apply. The cutoff is smooth_level times the total of the roughness measure; at each frequency the smoothing half-window is the largest one whose cumulative roughness stays at or below that cutoff. (The original papers describe stopping at the first window whose measure reaches the cutoff, which is one growth step later.)
method	Method used for smoothing; choices are "DZ" or "NZ".
digits	precision of output.
top	list top values
...	Other parameters.
x	periodogram

Details

The Kolmogorov-Zurbenko Periodogram is an estimate of the spectral density using the Kolmogorov-Zurbenko Fourier Transform (KZFT).

References

- I. G. Zurbenko, 1986: The spectral Analysis of Time Series. North-Holland, 248 pp.
- I. G. Zurbenko, P. S. Porter, Construction of high-resolution wavelets, Signal Processing 65: 315-327, 1998.
- A. G. DiRienzo, I. G. Zurbenko, Semi-adaptive nonparametric spectral estimation, Journal of Computational and Graphical Statistics 8(1): 41-59, 1998.
- R. Neagu, I. G. Zurbenko, Algorithm for adaptively smoothing the log-periodogram, Journal of the Franklin Institute 340: 103-123, 2003.
- Wei Yang and Igor Zurbenko, kzft: Kolmogorov-Zurbenko Fourier Transform and Applications, R-Project 2007.

See Also

[kzft](#), [kztp](#),

Examples

```
## Not run:
t<-1:6000
f1<-0.03
f2<-0.04
noise<-15*rnorm(length(t))
amp=1.5
s<-amp*sin(2*pi*f1*t)+amp*sin(2*pi*f2*t)
system.time(a<-kzp(s+noise,m=501,k=3))
b<-smooth.kzp(a, smooth_level=0.01)
par(mfrow=c(3,1))
plot(periodogram(s+noise),type='l')
plot(a)
plot(b)
par(mfrow=c(1,1))

# signal/noise
signal<-kzft(s+noise,m=501,k=3)
print(paste("signal-to-noise ratio = ", round(sqrt(var(2*Re(signal))/var(s+noise)),4) ))

summary(a, digits=2, top=2)

## End(Not run)
```

kzs	<i>Kolmogorov-Zurbenko Spline</i>
-----	-----------------------------------

Description

Kolmogorov-Zurbenko Spline

Usage

```
kzs(y,m=NULL,k=3,t=NULL)
```

Arguments

y	data
m	smooth. Passed to kzft , which requires an odd value; must be an odd integer when supplied explicitly.
k	The number of iterations for applying the KZFT
t	An indexing set

Details

Kolmogorov-Zurbenko Spline is essentially the Kolmogorov-Zurbenko Fourier Transform at the zero frequency.

References

- I. G. Zurbenko, The spectral Analysis of Time Series. North-Holland, 1986.
- I. G. Zurbenko, P. S. Porter, Construction of high-resolution wavelets, Signal Processing 65: 315-327, 1998.
- R. H. Shumway, D. S. Stoffer, Time Series Analysis and Its Applications: With R Examples, Springer, 2006.
- Derek Cyr and Igor Zurbenko, kzs: Kolmogorov-Zurbenko Spatial Smoothing and Applications, R-Project 2008.
- Derek Cyr and Igor Zurbenko, A Spatial Spline Algorithm and an Application to Climate Waves Over the United States, 2008 Joint Statistical Meetings.

See Also

[kzft](#),

Examples

```

n <- 1000
x <- (1:n)/n
true<-((exp(2.5*x)+sin(25*x))-1)/3

noise <- rnorm(n)
y <- true + noise

a<-kzs(y,m=61)

par(mfrow=c(2,1))
plot(y,type='l')
lines(true,col="red")

plot(a,type='l', ylim=c(-2,4))
lines(true,col="red")
par(mfrow=c(1,1))

#####
# second example
#####
t <- seq(from = -round(400*pi), to = round(400*pi), by = .25)
ts <- 0.5*sin(sqrt((2*pi*abs(t))/200))
signal <- ifelse(t < 0, -ts, ts)
et <- rnorm(length(t), mean = 0, sd = 1)
yt <- et + signal

b<-kzs(yt,m=401)
par(mfrow=c(2,1))
plot(yt,type='l')
lines(signal,col="red")

plot(b,type='l', ylim=c(-0.5,1))
lines(signal,col="red")
par(mfrow=c(1,1))

```

kzsv

Kolmogorov-Zurbenko Adaptive filter with Sample Variance.

Description

Sample variance of a Kolmogorov-Zurbenko adaptive filter. You want a sigma of at least 3.

Usage

```

kzsv(object)
## S3 method for class 'kzsv'
summary(object, digits=getOption("digits"), ...)
## S3 method for class 'kzsv'
plot(x, ...)

```

Arguments

object	The resultant object from kza function.
x	The results of the kza function.
digits	Precision of output.
...	Other parameters.

Examples

```
x <- c(rep(0,4000),rep(0.5,2000),rep(0,4000))
noise <- rnorm(n = 10000, sd = 1.0) # normally-distributed random variates
v <- x + noise
a<-kza(v, m=1000, k=3)
sv<-kzsv(a)
```

kztp

*Kolmogorov-Zurbenko Third-Order Periodogram***Description**

Kolmogorov-Zurbenko Third-Order Periodogram for estimating spectrums

Usage

```
kztp(x,m,k,box=c(0,0.5,0,0.5))
```

Arguments

x	The signal.
m	The window size for the kzft filter. Must be odd; KZFT's band-pass transfer-function properties are proven only for odd window lengths.
k	The number of iterations.
box	The window for the application of third-order periodgram.

Details

The Kolmogorov-Zurbenko Third-Order Periodogram is used to estimate spectral density of a signal. The smoothing methods are adaptive allowing the bandwidth of the spectral window to vary according to the smoothness of the underlying spectral density. For details, please see to DiRienzo and Zurbenko (1998) and Neagu and Zurbenko (2003).

References

I. G. Zurbenko, 1986: The spectral Analysis of Time Series. North-Holland, 248 pp. I. G. Zurbenko, P. S. Porter, Construction of high-resolution wavelets, *Signal Processing* 65: 315-327, 1998. W. Yang, I. G. Zurbenko, A semi-adaptive smoothing algorithm in bispectrum estimation, *Proceedings of the American Statistical Association*, Seattle, 2006. Wei Yang and Igor Zurbenko, *kzft: Kolmogorov-Zurbenko Fourier Transform and Applications*, R-Project 2007.

See Also[kzft](#), [kzp](#),**Examples**

```

t<-1:10000
y<-2*sin(2*pi*0.1*t)+3*sin(2*pi*0.2*t) + 10*rnorm(length(t))

a<-kztp(y,49,1)
z<-log(Mod(a))
#z<-smooth.kzp(z)

omega<-seq(0,1,length=51)[2:26]
#filled.contour(omega,omega,s,xlab="freq",ylab="freq",main="Smoothed 3rd Order Periodogram")

```

periodogram

*Periodogram***Description**

Raw periodogram.

Usage

```
periodogram(y)
```

Arguments

y The raw data.

Details

Periodogram is an estimate of the spectral density using FFT.

See Also[kzp](#),**Examples**

```

t<-1:1000
f1<-0.3
f2<-0.4
noise<-15*rnorm(length(t))
s<-3*sin(2*pi*f1*t)+3*sin(2*pi*f2*t)
plot(periodogram(s+noise),type='l')

```

rlv

*Rolling Variance Function***Description**

Rolling variance function, (rlv) will detect data variations of 1-dimensional vector, 2-dimensional matrix and/or 3-dimensional array, which in turn display their local discontinuities (boundaries).

Usage

```
rlv(inpt, krnl, pad = c("clamp", "zero"))
```

Arguments

inpt	input data.
krnl	length of the rolling window for detecting. Must be an odd number for the centered mode; krnl = 1 selects the legacy one-sided mode (the maximum over the corner windows that include the center cell).
pad	boundary policy. "clamp" (the default) clamps windows at the data edge and computes each variance from the cells actually present. "zero" reproduces the pre-4.2.0 behavior exactly: the data is conceptually zero-padded by the window radius, so windows near the border mix zeros into the variance, biasing it upward wherever the data is far from zero.

Details

The rlv is calculated based on a dynamic rolling window. Within a given window, directly estimating its variance allows to capture the local variations of the signal underlying analysis. Since this window is moving, when its size is optimal, the significant local variance atlas will be captured. It is neither necessary to make any assumption, nor to model anything, so it is a nonparametric statistic. However, direct application of rlv to data embedded in heavy background noises could fail to identify the significant local variations. In situations when the estimated data are embedded in background noise, necessary smoothness may be expected.

References

Igor G Zurbenko, Mingzeng Sun. Estimation of spatial boundaries with rolling variance and 2D KZA algorithm. Biometrics & Biostatistics International Journal. 7(4):263.270, 2018

Examples

```
## A. Signal/objects
# to create a cylinder
circleFun <- function(center = c(0,0),diameter = 1, npoints = 200){
  r = diameter / 2
  tt <- seq(0,2*pi,length.out = npoints)
  xx <- center[1] + r * cos(tt)
```

```

    yy <- center[2] + r * sin(tt)
    return(data.frame(x = xx, y = yy))
}
rxo1 <- circleFun(center = c(38, 38), diameter = 38, npoints = 200)
rxo1 <- circleFun(center = c(38, 38), diameter = 18, npoints = 200)
rxo2 <- circleFun(center = c(58, 58), diameter = 38, npoints = 200)
rxo2 <- circleFun(center = c(58, 58), diameter = 18, npoints = 200)
rxo3 <- circleFun(center = c(28, 78), diameter = 18, npoints = 200)
rxo3 <- circleFun(center = c(28, 78), diameter = 8, npoints = 200)

set.seed(3)
s2 <- matrix(rep(0,100*100),nrow=100)
for(i in 1:nrow(rxo1)) {
  s2[rxo1[i,2],38:rxo1[i,1]]=0.5
}
for(i in 1:nrow(rxo2)) {
  s2[rxo2[i,2],58:rxo2[i,1]]=1
}
# second cylinder
for(i in 1:nrow(rxo2)) {
  s2[rxo2[i,2],58:rxo2[i,1]]=0.5
}
for(i in 1:nrow(rxo3)) {
  s2[rxo3[i,2],28:rxo3[i,1]]=1
}
#third separated cylinder
for(i in 1:nrow(rxo3)) {
  s2[rxo3[i,2],28:rxo3[i,1]]=0.3
}
for(i in 1:nrow(rxo3)) {
  s2[rxo3[i,2],28:rxo3[i,1]]=0.6
}
signal=s2
## Graphing Library plotly
## Not run:
plot_ly(z=signal, type="surface")

## End(Not run)

## B. Rolling variance atlas

rolv=rlv(s2,3)
## Not run:
plot_ly(z=rolv, type="surface")

## End(Not run)

```

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